Online Appendix:
Food, Fuel and the Domesday Economy

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A Lengthy Calculations

A.1 Net power supplied

A.1.1 One crop

Energy rents are given by $\Delta - cr$, where r is the distance to the core. Marginal energy rents are given by $\Delta - cR = 0$ so that the maximum radius defining the exploitation zone is $R = \Delta/c$. The net power supplied to the core is:

$$\begin{aligned} W^S &= \int_0^{2\pi}\int_0^R v\Delta[1-\frac{c}{\Delta}v]dvd\varphi = 2\pi\int_0^R v\Delta[1-\frac{c}{\Delta}v]dv \\ &= 2\pi\left[\frac{v^2}{2}\Delta - c\frac{v^3}{3}\right]\Bigg|_0^R = \frac{\pi\Delta^3}{3c^2} \end{aligned} \tag{A.1}$$

A.1.2 Two Crops

The indifference point is calculated from:

$$p\left[\Delta_o - cr^*\right] = \left[\Delta_e - cr^*\right] \tag{A.2}$$

so that

$$r^* = \frac{R_o\left(p - \left(\frac{\Delta_e}{\Delta_o}\right)\right)}{(p-1)} \text{ where } R_o \equiv \Delta_o/c \tag{A.3}$$

The flow of (net) food energy they supply to the core is given by:

$$W_o^S = \int_0^{2\pi}\int_0^{r^*} v\left(\Delta_o - cv\right)dvd\varphi = \frac{\pi\Delta_o^3}{c^2}\left(\frac{p-\frac{\Delta_e}{\Delta_o}}{p-1}\right)^2\left(1 - \frac{2}{3}\frac{(p-\frac{\Delta_e}{\Delta_o})}{(p-1)}\right) \tag{A.4}$$

The flow of (net) fuel energy supplied over $r^* < r < R_e$ becomes:

$$W_e^S = \int_0^{2\pi}\int_{r*}^{R_e} v\left(\Delta_e - cv\right)dvd\varphi = \frac{\pi\Delta_e^3}{c^2}\left(\frac{1}{3} - \left(\frac{p\frac{\Delta_o}{\Delta_e}-1}{p-1}\right)^2\left(1 - \frac{2}{3}\frac{(p\frac{\Delta_o}{\Delta_e}-1)}{(p-1)}\right)\right) \tag{A.5}$$

For the area of exploitation, we are only interested in the external margin, which is simply the area of a circle with radius $\bar{R}_e$. Here we use the double integral notation, because it is useful for future calculations as we deviate from the simplest case.

$$EX = \int_0^{2\pi} \int_0^{\bar{R}_e} v dv d\phi = \pi \left(\frac{\Delta_e}{c} \right)^2 \tag{A.6}$$

A.2 Connected, Coastal and Edge Landscapes

We have thus far only considered simple landscapes, and it is tempting to think that many of our results are reliant on this assumption. Surprisingly, very similar and often exactly the same results hold for very different landscapes. To understand why, we construct a specific example to help build intuition. We show that a river through any core at location i, in landscape j effectively magnifies by a factor $(1 + g(\rho)) > 1$, the power densities of both energy sources. That is, we can write:

$$\Delta_e^j = (1 + g(\rho))^{1/3} \Delta_e^{j0} \; and \; \Delta_o^j = (1 + g(\rho))^{1/3} \Delta_o^{j0} \tag{A.7}$$

$$\begin{aligned} g(\rho) &= \frac{1}{\pi} \frac{\sqrt{1-\rho^2}}{\rho} - \frac{\bar{\theta}}{\pi} \geqslant 0 \\ c(\theta) &= \begin{cases} c\Gamma[\theta, \rho] & \text{if } \theta \leqslant \bar{\theta} \\ c & \text{if } \theta \geqslant \bar{\theta} \end{cases} \\ \Gamma[\theta, \rho] &= \left((1-\rho^2)^{1/2} \sin\theta + \rho \cos\theta \right) \end{aligned} \tag{A.8}$$

where the superscript $j0$ represents the landscape's original productivities without the river. Note $g(\rho)$ is increasing in the cost advantage of the river option $\rho \leqslant 1$. More efficient river transport implies $\rho \to 0$ which drives $g(\rho)$ towards infinity. $\bar{\theta} = \cos^{-1} \rho$ is independent of relative prices and uniquely determined by the cost advantage of the river option, ρ. This result is just a generalization of our finding of a transport multiplier, but now extended to a market economy with two energy sources.

Given the multiplicative role of $g(\rho)$, it is apparent the relative supply of food to fuel energy will be unaffected by the existence of the river and hence so too will be equilibrium relative prices. But since core income is homogeneous of degree three in power densities, we conclude that the existence of a river through the core raises real income by a factor $g(\rho) > 0$. It is then immediate that the resulting steady state population in a core with river transport is greater by a factor $g(\rho)$.

To calculate the number of such cores supported in a given landscape, we need to amend our earlier calculations for the area of exploitation coming from the extensive margin. In the case of a river, the extensive margin is given by

$$R(\theta) = \begin{cases} \frac{\Delta_e}{c\Gamma[\theta,\rho]} & \text{if } \theta \leqslant \bar{\theta} \\ \frac{\Delta_e}{c} & \text{if } \theta > \bar{\theta} \end{cases} \tag{A.9}$$

Therefore, the size of the exploitation zone for a river located core is given by:

$$EX^{river} = (1 + g(\rho)) \frac{\Delta_e^2}{c^2} \tag{A.10}$$

and this implies the number of cores supported in a landscape of size A is given by:

$$N = \frac{A}{EX^{river}} = \frac{Ac^2}{\Delta_e^2(1+g(\rho))} \tag{A.11}$$

Putting these results together we have found our previous predictions need only small amendments. The river works like a lowering of transport costs albeit of a specific type. It lowers transport costs, expands exploitation zones, and lowers settlement numbers. Its strength in doing so depends on the cost advantage as captured by ρ. The addition of the river enters simply and drives up the population within each settlement. Finally, since the total landscape population is found by multiplying settlement numbers by their average size, quite surprisingly we find the overall population in the landscape is independent of $g(\rho)$. This is a surprising result. Access to a river makes any core larger in terms of its population and lowers the number of settlements in any given area, but leaves the overall population size of the landscape unaffected. Rivers - just like the costs of transport we already studied - affect the organization of production but not its overall efficiency. They support larger centers since they lower transport costs, but if the efficiency of the underlying environment is unaffected by their presence then so too are population levels.

Although it is not immediately apparent, these results generalize quite nicely. For example, consider a landscape where a road intersected the core rather than a river. In this case our magnification effect would be $(1+2g(\rho))$ or almost twice as large since transport is made less costly in both directions. Settlement numbers fall, their size rises and our log linear equations need only the most obvious amendment. Similarly, consider a landscape where along one side there was an ocean and the other was landscape with a given power density. This is a coastal landscape. Assuming transport in both directions along an coast is possible (like our road case) then it should be apparent the exploitation zone for a core along the coast is just equal to $(1/2 + g_j(\rho_j))\frac{\Delta_e^2}{c^2}$. A seaside located core would have its population magnified by a factor $(1/2 + g(\rho))$ and the number of such locations in a given area has to fall as before. Next, consider a landscape completely filled with N cores all connected by a rectangular road network with two roads (North-South, and East-West) running through the cores. This is our connected landscape. Each of the cores would then be twice as connected as those connected with a single road and hence more populous in proportion to $(1+4g(\rho))$. This implies there would be fewer and larger cores, and again all adjustment in our predictions are straightforward.

Finally, consider a situation where the expansion in one (or more) directions from the core is blocked by either a geographical or political barrier. This is our edge landscape. In this situation, the landscape would be just half the population size of the simple landscape and it would have the same number of places.

The derivation details are in the subsections below.

A.2.1 Rivers

Energy rents in the case of a river are given by $w = \Delta - c(\theta)r$ where

$$c(\theta) = \begin{cases} c\Gamma[\theta,\rho] & \text{if } \theta \leqslant \bar{\theta} \\ c & \text{if } \theta \geqslant \bar{\theta} \end{cases} \tag{A.12}$$

and $\Gamma[\theta,\rho] = \left((1-\rho^2)^{1/2}\sin\theta + \rho\cos\theta\right)$. Farmers located at an angle $\theta < \bar{\theta}$ will deviate to the river. The cut-off radius for farmers located at an angle $\theta > \bar{\theta}$ is given by:

$$r_a = \frac{p\Delta_0 - \Delta_e}{(p-1)c} \tag{A.13}$$

The cut-off radius for farmers located at an angle $\theta < \bar{\theta}$ is given by:

$$r_b(\theta) = \frac{p\Delta_0 - \Delta_e}{(p-1)c\Gamma[\theta,\rho]} \tag{A.14}$$

The integrals of energy supplied for food and energy are given respectively by:

$$\begin{aligned}
W_o^{River} &= 2 \times \left[\int_0^{\bar{\theta}}\int_0^{r_b(\theta)} (\Delta_o - c\Gamma[\theta,\rho]v)v dv d\theta + \int_{\bar{\theta}}^{\pi}\int_0^{r_a} (\Delta_o - cv)v dv d\theta\right] \\
&= \frac{\Delta_o^3}{c^2}\left(\frac{p-\frac{\Delta_e}{\Delta_o}}{p-1}\right)^2\left(1-\frac{2}{3}\frac{p-\frac{\Delta_e}{\Delta_o}}{p-1}\right)\left(\pi-\bar{\theta}+\int_0^{\bar{\theta}}\Gamma[\theta,\rho]^{-2}d\theta\right) \\
&= \frac{\Delta_o^3}{c^2}\left(\frac{p-\frac{\Delta_e}{\Delta_o}}{p-1}\right)^2\left(1-\frac{2}{3}\frac{p-\frac{\Delta_e}{\Delta_o}}{p-1}\right)\left(\pi-\bar{\theta}+\frac{\sqrt{1-\rho^2}}{\rho}\right) \\
&= (1+g(\rho))W_o^S
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
W_e^{River} &= 2 \times \left[\int_0^{\bar{\theta}}\int_{r_b(\theta)}^{R_e(\theta)} (\Delta_e - c\Gamma[\theta,\rho]v)v dv d\theta + \int_{\bar{\theta}}^{\pi}\int_{r_a}^{R_e} (\Delta_e - cv)v dv d\theta\right] \\
&= \frac{\Delta_e^3}{c^2}\left(\frac{1}{3}-\left(\frac{p\frac{\Delta_o}{\Delta_e}-1}{p-1}\right)^2\left(1-\frac{2}{3}\frac{p\frac{\Delta_o}{\Delta_e}-1}{p-1}\right)\right)\left(\pi-\bar{\theta}+\int_0^{\bar{\theta}}\Gamma[\theta,\rho]^{-2}d\theta\right) \\
&= \frac{\Delta_e^3}{c^2}\left(\frac{1}{3}-\left(\frac{p\frac{\Delta_o}{\Delta_e}-1}{p-1}\right)^2\left(1-\frac{2}{3}\frac{p\frac{\Delta_o}{\Delta_e}-1}{p-1}\right)\right)\left(\pi-\bar{\theta}+\frac{\sqrt{1-\rho^2}}{\rho}\right) \\
&= (1+g(\rho))W_e^S
\end{aligned} \tag{A.16}$$

Now, we can write energy delivered to the core in the presence of a river as

$$W_o^{River} = (1+g(\rho))W_o^S \tag{A.17}$$

$$W_e^{River} = (1+g(\rho))W_e^S \tag{A.18}$$

where W_o^S and W_e^S are defined above, and $g(\rho)$ is given by the following expression:

$$g(\rho) = \frac{1}{\pi}\frac{\sqrt{1-\rho^2}}{\rho} - \frac{\bar{\theta}}{\pi}. \tag{A.19}$$

The area of the exploitation zone for a core with a river is given by:

$$EX^{River} = 2 \times \left[\int_0^{\bar{\theta}} \int_0^{R_e(\theta)} v dv d\theta + \int_{\bar{\theta}}^{\pi} \int_0^{R_e} v dv d\theta \right] \tag{A.20}$$

$$= \frac{\Delta_e^2}{c^2} \left[\pi - \bar{\theta} + \int_0^{\bar{\theta}} \frac{1}{\Gamma(\theta)^2} d\theta \right] \tag{A.21}$$

$$= (1 + g(\rho)) EX \tag{A.22}$$

A.2.2 Roads

Next consider a core with a road instead of a river. A road is a river that allows for lower costs of transportation in both directions. This means the energy supplied is now given by:

$$W_e^{Road} = 4 \times \left[\int_0^{\bar{\theta}} \int_{r_b(\theta)}^{R_e(\theta)} (\Delta_e - c\Gamma[\theta, \rho] v) v dv d\theta + \int_{\bar{\theta}}^{\pi/2} \int_{r_a}^{R_e} (\Delta_e - cv) v dv d\theta \right]$$

$$W_e^{Road} = (1 + 2g(\rho)) W_e^S \tag{A.23}$$

$$W_o^{Road} = 4 \times \left[\int_0^{\bar{\theta}} \int_0^{r_b(\theta)} (\Delta_o - c\Gamma[\theta, \rho] v) v dv d\theta + \int_{\bar{\theta}}^{\pi/2} \int_0^{r_a} (\Delta_o - cv) v dv d\theta \right]$$

$$= (1 + 2g(\rho)) W_o^S \tag{A.24}$$

The area of the exploitation zone for a core with a road is given by:

$$EX^{Road} = (1 + 2g(\rho)) EX \tag{A.25}$$

A.2.3 Connected Landscape

Consider two roads crossing the core perpendicular to each other. Under the simplifying assumption that $\bar{\theta} < \pi/4$ we get:

$$W_e^{Connected} = (4g(\rho) - 3) W_e^S \tag{A.26}$$

$$W_o^{Connected} = (4g(\rho) - 3) W_o^S \tag{A.27}$$

If $\bar{\theta} > \pi/4$ then we have to modify the $g(\rho)$ function to account for a small change. Define $\tilde{g}(\rho)$ as

$$\tilde{g}(\rho) = \frac{1}{\pi} \frac{\sqrt{1 - \rho^2}}{\rho} \tag{A.28}$$

and in this case we find

$$W_e^{Connected} = (1 + 4\tilde{g}(\rho)) W_e^S \tag{A.29}$$

$$W_o^{Connected} = (1 + 4\tilde{g}(\rho)) W_o^S \tag{A.30}$$

The area of the exploitation zone of a city in a connected landscape with $\bar{\theta} < \pi/4$ is given by

$$EX^{Connected} = (1 + 4g(\rho))EX \tag{A.31}$$

and for $\bar{\theta} > \pi/4$, it is equal to

$$EX^{Connected} = (1 + 4\tilde{g}(\rho))EX \tag{A.32}$$

A.2.4 Coastal

The energy provided to the core of a coastal city is given by

$$W_e^{Coastal} = (1/2 + g(\rho))W_e^S \tag{A.33}$$

$$W_o^{Coastal} = (1/2 + g(\rho))W_o^S \tag{A.34}$$

The area of the exploitation zone for a city in a coastal landscape is given by

$$EX^{Coastal} = (1/2 + g(\rho))EX \tag{A.35}$$

A.2.5 Edge

The energy provided to the core of a coastal city is given by

$$W_e^{Edge} = \frac{1}{2}W_e^S \tag{A.36}$$

$$W_o^{Edge} = \frac{1}{2}W_o^S \tag{A.37}$$

The area of the exploitation zone of a city in an Edge landscape is given by

$$EX^{Edge} = \frac{1}{2}EX \tag{A.38}$$

A.3 Political Aggregates

A.3.1 Aggregate income

Aggregate income is the value sum of all energy rents. Using the general equilibrium price p, we have for the simple landscape:

$$\begin{aligned} I(p, \Delta_o, \Delta_e) &= pW_o + W_e \\ &= \frac{\pi\Delta_e^3}{3c^2}\left(1 + \frac{(p\frac{\Delta_o}{\Delta_e} - 1)^3}{(p-1)^2}\right) \end{aligned} \tag{A.39}$$

Using this result, we can calculate aggregate income for all other landscapes as:

$$I(p, \Delta_o, \Delta_e) = pW_o^\ell + W_e^\ell \tag{A.40}$$

where $\ell \in \{Connected, Coastal, Edge\}$. Using the results from above, we obtain

$$\begin{aligned} I^{Connected}(p, \Delta_o, \Delta_e) &= (1 + 4g(\rho))I(p, \Delta_o, \Delta_e) \\ I^{Coastal}(p, \Delta_o, \Delta_e) &= (1/2 + g(\rho))I(p, \Delta_o, \Delta_e) \\ I^{Edge}(p, \Delta_o, \Delta_e) &= \frac{1}{2}I(p, \Delta_o, \Delta_e) \end{aligned} \tag{A.41}$$

A.3.2 Average population size

The cities belonging to different landscapes will be characterized by a population size that is proportional to the cube of the productivity of the landscape. In particular, for the simple landscape, the size of a city is given by:

$$L_{ij} = MI(p, \Delta_o, \Delta_e)/\beta(p) = M\frac{\pi\Delta_e^3}{3c^2}\left(1 + \frac{(p^{\frac{\Delta_o}{\Delta_e}} - 1)^3}{(p-1)^2}\right)/\beta(p) \tag{A.42}$$

where $\beta(p)$ is the appropriate price index. From here it follows directly from the results above that

$$\begin{aligned} L_{ij}^{Connected} &= (1 + 4g(\rho))L_{ij} \\ L_{ij}^{Coastal} &= (1/2 + g(\rho))L_{ij} \\ L_{ij}^{Edge} &= \frac{1}{2}L_{ij} \end{aligned} \tag{A.43}$$

A.3.3 Number of places

The number of places in a landscape is calculated by dividing the area of the landscape, A_j, but the area of the exploitation zone. In particular, for the simple landscape, the number of places is given by:

$$N = \frac{A_j}{\pi\left(\frac{\Delta_e}{c_j}\right)^2} = \frac{A_j c_j^2}{\pi}\Delta_e^{-2} \tag{A.44}$$

Using this result we, we can write

$$N^\ell = \frac{A_j}{EX^\ell} \tag{A.45}$$

where $\ell \in \{River, Connected, Coastal\}$ to find

$$\begin{aligned} N^{Connected} &= (1 + 4g(\rho))^{-1}N \\ N^{Coastal} &= (1/2 + g(\rho))^{-1}N \\ N^{Edge} &= 2N \end{aligned}$$

A.3.4 Total population landscape

Total population for the landscape is given by the multiplication of the number of places and the size of each place. For the simple landscape this is equivalent to:

$$L_j = NL_{ij} = \frac{1}{3}A_j M\Delta_e \left(1 + \frac{(p\frac{\Delta_o}{\Delta_e} - 1)^3}{(p-1)^2}\right)/\beta(p) \tag{A.46}$$

which is independent of the geography of the landscape.

B von Thünen and Samuelson

B.1 Linear transportation costs

Work is equal to force, F, times distance, x, or work is $W_k = F \cdot x$. Force is in turn equal to mass, M, times acceleration g; as any mass moved horizontally must overcome the force of gravity as mediated by friction in transport, where μ is the coefficient of friction.[1] This work is done per unit time since power is a flow (as is for example the flow of labor and capital services that creates the flow of useful output in standard analyses). Choosing units is inconsequential and if we measure time in seconds, then the flow of work, W_k, measured in Joules per second is now power requirements measured in Watts.[2]

Now consider the costs of moving energy resources providing one Watt of power just one meter. Consider an energy source with power density Δ [Watts/m^2] with a concentration measure d[kg/m^2], this in turn implies the energy resources it represents must weigh d/Δ kilograms. Moving this mass one meter, and overcoming friction, requires a flow of power of $\mu gd/\Delta$. Therefore, μgd is the number of Watts needed to transport Δ Watts worth of an energy source with power density Δ, one meter. When energy sources are transported by land we have $c = \mu gd$. In this case, the zero energy margin distance is $R^* = \frac{\Delta}{\mu gd}$ and total energy collected is given by $W^* = \frac{\pi\Delta^3}{(\mu gd)^2}$.

B.2 Iceberg costs

Our model generates results much like those in von Thünen's Isolated State (1826). Our costs are constant per unit mass and thus generate a limit to the exploitation zone. We derive a primitive per meter transport cost similar to that assumed by von Thünen and but now explicitly linked to fundamentals such as the energy density of resources, the difficulty of terrain and the physical amount of work required to transport objects (Moreno-Cruz and Taylor, 2018). Just like von Thünen, we solve for a circular area of exploitation whose margins are determined by a zero rent condition. While von Thünen determined the zero

[1] We are ignoring static friction encountered when the object first moves. The force that needs to be overcome to keep an object in motion is equal to the normal force times the coefficient of friction. Since the object is moving horizontally, the normal force is just gravity times the mass of the object . The coefficient of friction is a pure number greater than zero; and force is measured in Newtons.

[2] Expending 1 Joule of energy in 1 second means you are delivering 1 Watt of power.

rent condition based on transportation cost only, we find that limit is a function of both productivity and transportation costs.

von Thünen motivates his constant per unit mass costs by assuming transportation costs are measured in "natural units," using the analogy that the horse eats the grain it is transporting to the city (Clark 1967). In our case, we use energy to move energy. Samuleson (1983) takes this analogy to justify his assumption about iceberg transportation costs. While the assumption of iceberg costs simplifies some of the calculations, Samuelson gets in trouble when trying to identify a limit to the exploitation. Under the iceberg assumption, the exploitation zone of the outer ring would extent infinitely.[3]

In Samuelson (1983)'s model, labor is required in situ to produce agricultural output and he shows that in equilibrium population density is declining away from the city core. Samuelson's limit on the exploitation zone comes from the assumption that labor is too scarce at long distances from the core, and thus at some point, more resources will not increase total energy delivered to the core.

Reading further von Thünen, Clark (1967) argues the idea of "natural units" captures all costs required to transport good, measured in terms of the good being transported, including paying for the means of transportation (the wagon) or the fact that waste of burning energy for transportation (incomplete combustion) leaves mass that needs to be transported. Introducing fixed costs the naturally generates a limit in the exploitation zone.[4]

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[3]To see how our model generates Samuleson's result, assume there is a farmer located at a distance r away from the city center. This farmer spends some of the energy he collects in transportation. Energy used in transportation per unit of distance is

$$W^T(x) = \frac{c}{\Delta} W(x)dx \tag{B.1}$$

where $W(x)$ is the amount of energy that is left after transporting the original amount of energy a distance x. That is, energy at distance $W(x + dx) = W(x) - \frac{c}{\Delta}W(x)dx$. We can rewrite this as

$$\frac{W(x + dx) - W(x)}{dx} = -\frac{c}{\Delta} W(x) \tag{B.2}$$

In the limit where dx approaches zero, the lefthand side in the previous equation becomes, by definition, dW/dx. The following differential equation determines the energy surplus at distance x:

$$\frac{dW(x)}{dx} = -TC(W(x)) = -\frac{c}{\Delta} W(x) \tag{B.3}$$

Assume the farmer is sitting in 1m^2 of land with power density Δ so that $W(0) = \Delta$. The solution to this differential equation is:

$$W(x) = \Delta e^{-\frac{c}{\Delta}x} \tag{B.4}$$

If we evaluate this expression at distance r, then we obtain the energy delivered to the city by a farmer located a distance r away from the core. In this case, due to the exponential nature of the energy supplied, there is not a limit to how far farmer can be located away from the city as the amount of energy brought into the city is always positive.

[4]Assume transportation costs are given by:

$$W^T(x) = \left(C(W_0) + \frac{c}{\Delta} W(x)\right) dx \tag{B.5}$$

where $C(W_0)$ are the fixed costs incurred to move energy W_0. Total energy at distance $W(x + dx) =$

C Data Appendix

Our data come from one of the most interesting records in history. The Domesday book is a historical artifact, but also an economic and cultural one. In our paper, we use it for its value as an accounting document. In this appendix, we describe how we handle the data and the assumptions we made to be able to use it as an input in our paper.

C.1 Domesday Book

The data used in this project was obtained from the History Data Service and is the Electronic Edition of Domesday Book: Translation, Databases and Scholarly Commentary, located at https://www.ukdataservice.ac.uk. The data come in many tables and observations are organized and traced throughout using the identifier "structidx." In the file "ByPlaces.csv" we have information on the "4-figures OS coordinates of Domesday vill" of the places, or settlements, referenced in the Domesday book. We use these coordinates to add environmental and landscape information, as we discuss below. We also have a shapefile of the parishes in England that match very closely to the location of places. We can then use this shapefile to calculate the approximate area of the place at the time. We then aggregate from Parishes to Hundreds and add circuit and county identifiers.

Most of the census information is in the file "Manors.csv." We use the following information:

Population data: We obtain the population data at the place as the sum of the population entries in the Domesday book "Villager + Smallholders." We aggregate them at the parish level and the hundred level.

Income data: We obtain income data at the place from the variables "Value 86" and "Value 66" which are the "value of the holding to its lord in 1086" and "value of the holding to its lord in 1066" respectively.

Ploughteams: We obtain income data from the variable "TotalPloughs" which captures "total number of ploughteams attributed to the holding, the teams each assumed to comprise 8 oxen." An oxgang captures how much land was tillable by one ox in a ploughing season. One ploughteam, or carucate, was equivalent to 120 acres, but there was substantial variance across villages. We use ploughteams to calculate the abundance of arable land in a hundred.

$W(x) - \left(C(W_0) + \frac{c}{\Delta} W(x)\right) dx$. Rearranging terms we can rewrite this expression as

$$\frac{W(x+dx) - W(x)}{dx} \equiv \frac{dW(x)}{dx} = -\left(C(W_0) + \frac{c}{\Delta} W(x)\right) \tag{B.6}$$

The solution to the differential equation is:

$$W(x) = \left(W_0 + \frac{\Delta}{c} C(W_0)\right) e^{-\frac{c}{\Delta} x} - \frac{\Delta}{c} C(W_0) \tag{B.7}$$

Define R as the radius for which $W(R) = 0$; that is, energy supplied to the city by sources farther than R is zero. The solution for R is:

$$R = \frac{\Delta}{c} \ln\left(\frac{c}{\Delta} \frac{W_0}{C(W_0)} + 1\right) \tag{B.8}$$

C.2 Environmental and Landscape Data

Ruggedness: We obtain terrain ruggedness from Nathan Nunn and Diego Puga article 'Ruggedness: The blessing of bad geography in Africa', published in the Review of Economics and Statistics 94(1), February 2012: 20-36. They provide the Terrain Ruggedness Index (in milimetres) at the level of individual cells on a 30 arc-seconds grid across the surface of the Earth.
Temperature: We use data from FAO-GAEZ (http://gaez.fao.org/Main.html). Our data is baseline mean annual temperature data for the years 1960-1990 measured in ^{o}C.
Precipitation: We use data from FAO-GAEZ (http://gaez.fao.org/Main.html). Our data is baseline mean annual precipitation data for the years 1960-1990 measured in mm.
Landscapes: We obtain data on navigable rivers and ancient (major and minor) roads from Satchell, M., Shaw-Taylor, L., Wrigley, E. (2018) [data collection]. UK Data Service. SN: 852999, http://doi.org/10.5255/UKDA-SN-852999. We use ArcGIS to identify coastal counties and counties bordering Wales. The approximate location of Norman castles is also in our database. All these variables are creating using a buffer of 2km around the feature. For example, the variable Rivers counts the number of parishes that are less than 2 km away from a navigable river. The same procedure applies to the other landscape characteristics such a road landscapes, coastal landscapes, and edge (Wales) landscapes.

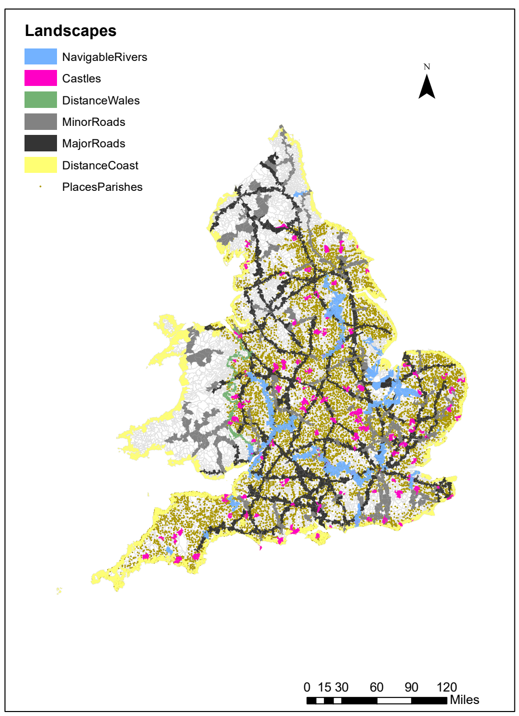

Figure C.1: Landscapes: This map summarizes the distribution of landscapes across England and Wales.

C.3 Variables construction and summary statistics

The summary statistics of the variables used in our analysis are presented in Table C.1. Below, we show how we constructed each variable. There are 875 hundreds in our sample.
No. Settlements is the count of settlements withing a Hundred.
Population Settlement is calculated as the area-weighted average of the settlement population. That is,

$$\text{Pop. Sett.} = \sum_{i=1}^{N_j} \text{Population}_{ij} * \frac{\text{Area}_{ij}}{Area_j} \tag{C.1}$$

Total Population at the hundred level is the sum of the populations in all settlements within a hundred.
Income is calculated via an area-weighted average of the income of the place as in equation (C.1).
Abundance of arable land is calculated as the total number of ploughteams, divided by the area of the hundred.

The environmental and terrain variables *ruggedness, temperature and precipitation* are represented as weighted averages and calculated the same way as in equation (C.1). Aggregate landscape characteristics are calculated as the count of parishes that belong to a particular landscape in a hundred. If two parishes have rivers, then the River variables at the hundred level is 2. We then divide by the total number of parishes in the hundred to get at a measure of intensity.

Table C.1: Summary Statistics

	Mean	Std. Dev.	Min	Max
No. Settl.	21.04	21.76	1.00	151.00
Pop. Settl.	21.69	26.66	0.00	344.15
Total Population	288.98	408.45	0.00	6801.00
Income 1086	8.89	12.45	0.00	142.77
Abund. Arable Land	0.96	1.99	0.00	34.27
Area	225.17	244.45	0.55	1553.64
Ruggedness	20756.88	19143.81	138.40	159844.53
		Environmental Variables		
Temperature	9.59	0.48	7.10	10.67
Precipitation	726.40	146.38	560.34	1285.02
		Landscapes		
Coastal	0.10	0.25	0.00	1.00
Rivers	0.07	0.20	0.00	1.00
Minor Roads	0.07	0.19	0.00	1.00
Major Roads	0.19	0.28	0.00	1.00
Border Wales	0.01	0.06	0.00	1.00
Castles	0.07	0.20	0.00	1.00

Notes: Summary statistics for the dependent and independent variables used in our analysis.

Figure C.2: Spatial distribution of dependent and independent variables

D Instrumental Variables Approach

We are concerned with the potential endogeneity of income and our outcomes of interest. In our model income is determined independently of labor, and is not affected by the number of neighboring settlements via trade or migration. In other settings it would or could be determined by these factors, contaminating the errors in our OLS specification. To address this issue we need an instrumental variable with across hundred variation that is both correlated with incomes in 1086, and otherwise excluded from the regressions. We instrument contemporaneous income levels in 1086 using incomes reported in 1066. The basic idea is that the cross-sectional distribution of incomes in 1066 contains information about the cross-sectional distribution of hundred productivities that is, in turn, responsible for an exogenous component of variation in the 1086 values. The logic behind this instrument comes from considering an alternative model where labor enters the national income function. For example, consider a neoclassical model where there are many hundreds that differed only in terms of their Hicks-neutral technological productivity. Then labor demands would differ across the hundreds since they differ in productivities, but the Malthusian steady state supply of labor is perfectly elastic at the subsistence wage. The hundreds would differ in terms of population sizes, and incomes as a result. If 1066 is an existing steady state, the variation we see across hundreds in terms of their average settlement incomes is perfectly correlated with their cross-sectional differences in productivity. Our underlying assumption is that the cross-sectional distribution of income in 1066 affects outcomes in 1086 only through the information it contains in terms of hundred productivities that is captured in 1086 average hundred incomes. Further complications arise if the observations in 1066 were not from an existing steady state; if our model is not true so labor supply is not elastic at the subsistence wage; or if transitory shocks from 1066 reverberated through to other variables in 1086 as well. To deal with any of these issues, a panel data estimation would be required, and of course we have no ability to generate such a panel as the 1066 entries in the Domesday book are very incomplete. With these caveats in mind consider Table D.1.

Table D.1 shows the strength and quality of our instruments. We restrict the sample the hundreds that report income in 1066. The number of observations is reduced from 875 to 712 hundreds. In Panels A through C, the first column shows the regression results for the Number of Settlements, the second column for Settlement Size and the third column for Total Population. To save space, we indicate but don't report when we are using Landscape controls (Coastal, Rivers, Minor Roads, Major Roads, Border Wales, Castles and their interactions) and Environmental Controls (Temperature and Precipitation).

Panel A replicates our results for the OLS from the main paper but with the restricted sample. Restricting the sample changes the point estimates, but their remain statistically significant, of equal magnitude and sign.

Panel B shows a statistically significant relation between income 1066 and the outcome variable. Higher income in 1066 is related with a lower number of settlements, higher settlement populations and higher overall population. These results are all statistically significant at the 5% level.

In Panel D the dependent variable is Income in 1086 and shows the first stage results. A positive relation between income in 1066 and income in 1086 exists across all specifications. this relations is statistically significant at the 1% level. The first stage Kleibergen-Paap

F-statistic for the excluded instrument is 12.5 or higher across different regressions. This implies it is unlikely that our estimates are biased by weak instruments.

Panel C shows the 2SLS results. In all cases, our earlier results are strengthened by instrumenting. The overall take away from our analysis remains the same.

Table D.1: Instrumental Variables (Value 86=Value 66)

	No. Settlements	Settlement Size	Total Population
		Panel A: OLS	
Income	-0.282^a	0.812^a	0.448^a
	(0.039)	(0.042)	(0.081)
Abund. Arable Land	0.745^a	0.268^b	1.276^a
	(0.115)	(0.085)	(0.142)
Area	0.769^a		0.923^a
	(0.017)		(0.050)
Ruggedness	0.010		0.106
	(0.037)		(0.069)
Constant	-0.784^b	5.499	8.362^c
	(0.317)	(2.870)	(3.851)
		Panel B: Reduced Form	
Income 1066	-0.172^a	0.414^b	0.175
	(0.046)	(0.156)	(0.108)
Abund. Arable Land	0.561^a	0.805^a	1.653^a
	(0.134)	(0.168)	(0.149)
Area	0.757^a		0.951^a
	(0.017)		(0.048)
Ruggedness	0.011		0.111
	(0.027)		(0.084)
Constant	-0.803^a	8.854^b	10.523^b
	(0.208)	(2.942)	(3.543)
		Panel C: 2SLS	
Income	-0.340^a	0.801^a	0.347^a
	(0.040)	(0.071)	(0.113)
Abund. Arable Land	0.813^a	0.279^a	1.396^a
	(0.087)	(0.104)	(0.149)
Area	0.775^a		0.933^a
	(0.015)		(0.046)
Ruggedness	0.012		0.109^c
	(0.035)		(0.064)
Constant	-0.780^b	5.646^b	9.480^a
	(0.323)	(2.753)	(3.636)
		Panel D: First Stage — Dependent Variable: Income 1086	
Income 1066	0.507^a	0.516^a	0.504^a
	(0.149)	(0.149)	(0.144)
Abund. Arable Land	0.740^a	0.656^a	0.741^a
	(0.137)	(0.124)	(0.128)
Area	0.054^a		0.052^a
	(0.016)		(0.016)
Ruggedness	0.002		0.006
	(0.042)		(0.040)
Constant	0.066	4.003	3.005
	(0.534)	(5.501)	(4.889)
Landscapes	X	X	X
Environment		X	X
County	X	X	X
Observations	712	712	712

Notes: All variables are log transformed using $\tilde{x} = ln(1+x)$. Errors clustered at the circuit level are reported in round parentheses. ^{a}, ^{b}, and ^{c} denote significance at the 1 , 5 and 10. All regression include landscapes and environmental variables when required.