

Online appendix to accompany

“An Energy-centric Theory of Agglomeration”

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A The Energy Industry

The aggregate 10 trillion dollar number is an estimate calculated as follows: annual energy consumption was 12,807.1 million tons of oil equivalent in 2013 (British Petroleum, 2015, 40). A ton of oil is equivalent to 7.33 barrels, and the average spot price of a barrel of Brent crude oil, a common benchmark for the world price of oil, was \$108.56 (US) in 2013 (Energy Information Administration, 2015). Calculating annual global energy sales in 2013 using this information yields an estimate of \$10.191 trillion US.

Global energy consumption in barrels of oil equivalent per second in 2013 is calculated by once again taking annual energy consumption of 12,807.1 million tons of oil equivalent, multiplying by 7.33 barrels of oil per ton and then dividing that figure by $31,536,000 = 60 \times 60 \times 24 \times 365$ seconds in 2013 to get 2976.79 barrels of oil equivalent consumed per second.

The World Trade Organization valued 2013 global fuels exports (f.o.b.) at \$3.258 trillion US (2014, 62). Fuels are classified by the United Nations Statistics Division (2006) in section 3 of the Standard International Trade Classification, Rev. 4 as: coal, coke and briquettes; petroleum, petroleum products and related materials; gas, natural and manufactured; and electric current.

Data on global pipeline infrastructure listed by country is accessible through the Central Intelligence Agency’s World Factbook (2013). Simple addition yields global oil and gas pipeline infrastructure totalling 3,573,235 km, with natural gas pipelines accounting for 2,855,017 km. Oil, refined products and liquid petroleum gas pipelines account for the remaining 718,218 km.

The global oil tanker fleet has a total capacity of 472.890 million deadweight tons (DWT), corresponding to a 29.1 percent share of the total capacity of the global merchant fleet. Liquefied natural gas (LNG) carriers have a combined total capacity of 44.346 million DWT, a 2.7 percent share of the deadweight tonnage of the global merchant fleet. Combining these numbers yields a total global oil and gas merchant vessel capacity of 517.236 DWT. All information on capacity and percentage share of global fleet capacity is taken from the United Nations Conference on Trade and Development (2014, 29).

Fortune Magazine's Global 500 list (2013) ranks the largest companies in the world, by revenue. Included in the list's top 10 are energy companies Royal Dutch Shell, Exxon Mobil, Sinopec Group, China National Petroleum, British Petroleum, State Grid and Total.

B von Thunen and Iceberg Costs of Transport

The transport cost assumptions adopted in von Thunen are subtly, but importantly, different from what we have assumed here. In short, iceberg costs require the energy costs of transportation to fall immediately and completely as energy is expended. At the practical level this rules out containers for fuel storage or combustion, residues left from incomplete combustion, and no mass of the vehicle carrying the load. In terms of the oat eating horse example both Von Thunen and Samuelson used, the horse cannot have any mass, the oats cannot remain resident in the horse, and there is of course no wagon to pull. It is fair to say that while the iceberg assumption is tractable it is also a knife edge assumption as we will show below. If even an epsilon of the mass of energy is wasted in moving containers, in moving engines or left in incompletely combusted particles then the transport process produces a result significantly different than von Thunen's but qualitatively the same as in our specification. Specifically it will lead to a formulation where there is a maximum zone of exploitation tied to the power density of energy.

To be precise, consider the case of renewable with mass discussed in section 2.1.1 on the

main text. Assume there is some fixed cost associated with moving energy a distance dx . Then transportation costs over this increment are given by:

$$W^T(x) = \left(C(W_0) + \frac{\mu g d}{\Delta} W(x) \right) dx$$

Total energy remaining at distance $x+dx$ is given by $W(x+dx) = W(x) - (C(W_0) + \frac{\mu g d}{\Delta} W(x)) dx$. Rearranging terms we can rewrite this expression as

$$\frac{W(x+dx) - W(x)}{dx} = \frac{dW(x)}{dx} = - \left(C(W_0) + \frac{\mu g d}{\Delta} W(x) \right)$$

The solution to this differential equation is:

$$W(x) = \left(W_0 + \frac{\Delta}{\mu g d} C(W_0) \right) e^{-\frac{\mu g d}{\Delta} x} - \frac{\Delta}{\mu g d} C(W_0)$$

Define R as the radius for which $W(R) = 0$; that is, the energy supplied to the core by any energy source further away than R is zero. The solution for R is:

$$R = \frac{\Delta}{\mu g d} \ln \left(1 + \frac{\mu g d}{\Delta} \frac{W_0}{C(W_0)} \right)$$

The iceberg assumption occurs when $C(W_0) = 0$ since in this case R goes to infinity. The case we consider in the text arises when $C(W_0)$ is proportional to the mass of energy transported; specifically that $C(W_0) = \frac{\mu g d}{\Delta} \frac{W_0}{e-1}$ since then we obtain $R = \frac{\Delta}{\mu g d}$.

Two observations are in order. First, there exists a finite margin of exploitation for any $C(W_0) > 0$. Thus, iceberg costs represent a knife edge assumption; any value other than $C(W_0) = 0$ generates a qualitatively different result. Any and all energy sources sharing the same - infinite - margin of exploitation when $C(W_0) = 0$; they have finite and different margins of exploitation for any $C(W_0) > 0$. Second, iceberg costs have proven tractable in general equilibrium models because they allow us to model the transportation system without

introducing another economic activity complicating predictions in small dimensional models. The formulation in the body of the paper does however respect this constraint. Note the only costs of transport come from moving energy (and not containers, equipment or engines even though the result is consistent with formulations with these fixed costs), the key to our result is our assumption that the mass of energy is transported even as it is used (converted) in transport.

C Extensions

The model presented in the paper is decidedly stark and abstract. In this section of the Online Appendix we present several extensions to showcase the versatility and wider applicability of the Only Energy model. In the paper we treat power density as a primitive. While this is true in principle, investments can be made to improve the quality of the resources or to reduce the costs of transporting them to the core. In the first extension we show the incentives to expand the exploitation zone and to upgrade the quality of the resources; more importantly we show that these incentives are stronger when the intrinsic quality of the resources is higher. Another important assumption in the paper is that power density is distributed uniformly across space. In our second extension we consider the case where the distribution of resources is patchy or punctiform and we also introduce the possibility of uncertain location of resources across space. We show that all the results we find using the uniform distribution hold under these alternative resource distributions. We then introduce the case of non-renewables. Our purpose here is to demonstrate how spatial productivity is germane to both renewables and non-renewables although the details are in some cases importantly different. Perhaps more importantly we demonstrate how spatially productive environments lead to a bunching of resource extractions in calendar time.

C.1 Resource Upgrading and Endogenous Exploitation Zone

C.1.1 Energy Investments

The magnification effect (Proposition 2 in the main text) tells us two quite useful things. The first is that access to low cost transport options is equivalent to raising the power density of surrounding resources. Given this equivalence, for any investment lowering the physical cost of transport there is an equivalent one raising the power density of energy resources by upgrading. Upgrading resources and investing to lower transport costs are opposite sides of the same coin.

The second is that the marginal impact of low cost transport options is greater for more power dense resources. Simply put, the energy supply consequences of say river access is far higher in a world run by, for example, coal than it would be in one run on biomass. This suggests that investments to lower transport costs will be greatest in situations where energy resources are already quite power dense.

To examine the motivation for energy investments assume the cost of building and maintaining transport that delivers an efficiency of ρ is given by $h(\rho)$. Assume $\rho > \bar{\rho}$ where $\bar{\rho}$ is the minimum physically possible value of the coefficient of friction. Since a lower value of ρ implies lower cost energy transport we assume $h(\rho)$ is a decreasing and convex function: $h'(\rho) < 0$, $h''(\rho) > 0$ with $h(1) = 0$ and $h'(1) < 0$. Then energy supplied net of infrastructure costs are $W^N = W^S(\rho) - h(\rho)$. For concreteness consider

$$W^S(\rho) = \frac{1}{3} \frac{\Delta^3}{c^2} g(\rho) \tag{C.1}$$

$$g(\rho) = \pi - 2\bar{\theta} + 2 \int_0^{\bar{\theta}} ((1 - \rho^2)^{1/2} \sin \theta + \rho \cos \theta)^{-2} d\theta \geq 0 \tag{C.2}$$

as in the paper and we are improving river transportation by dredging, locks, canals, main-

taining ports etc. The optimal investment problem is simply given by

$$\max_{\rho} W^N = W^S(\rho) - h(\rho) = \frac{1}{3} \frac{\Delta^3}{c^2} g(\rho) - h(\rho) \quad (\text{C.3})$$

When the solution is interior, the first order condition that maximizes energy requires

$$\frac{1}{3} \frac{\Delta^3}{c^2} g'(\rho) = h'(\rho) \quad (\text{C.4})$$

The left hand side of this equation is the marginal benefit from improved transport and it is again a cubic in power density showing a strong relationship between power density and the marginal benefit of further investments. The right hand side is simply marginal costs of improved transportation. Since marginal benefits are bounded and marginal costs of the first unit of investment are positive, with sufficiently low power density no investments in transport improvements will occur. In situations with higher power density an interior solution will obtain and we can write the implicit solution to (C.4) as $\tilde{\rho}(\Delta)$. Straightforward differentiation now shows¹

$$\frac{d\rho}{d\Delta} = -\frac{\rho}{\Delta} \frac{3}{\epsilon'_{h\rho} - \epsilon'_{g\rho}} < 0$$

where $\epsilon'_{h\rho} = -\rho h''(\rho)/h'(\rho)$ and $\epsilon'_{g\rho} = -\rho g''(\rho)/g'(\rho)$ and the second order conditions sign the expression. Therefore, we have proven:

Proposition C.1 *Complementarity.* *There exists a critical level of power density $\Delta^c > 0$ such that for energy resources with $\Delta < \Delta^c$ no investments in cost reducing transport occur; but for environments with $\Delta > \Delta^c$, investment is positive and more power dense energy resources call forth greater investments in cost reducing transportation investments.*

Proposition C.1 links the incentive for transport and upgrading of resources to their power density. Resources with low power density will neither be upgraded nor call forth

¹We have already assumed $h''(\rho) > 0$; it can be shown that $g(\rho)$ is convex for all values of ρ . The second order conditions required for a maximum imply $\frac{1}{3} \frac{\Delta^3}{c^2} g''(\rho) < h''(\rho)$ which simply says the costs are more convex than the benefits from investing in infrastructure.

large investments to improve transportation. Biomass for example is not a very power dense energy resource, and in a world run on biomass we should expect very local energy markets that are severely constrained by transport costs. These local markets could be expanded somewhat by simple upgrading (charcoal), but the incentives for dedicated investments should be weak. With the advent of fossil fuels new incentives arose. More power dense fuels like coal brought forth large investments in lowering transport costs and upgrading. Canals, railroads, and upgrading coal to coke could be seen as endogenous responses to its higher power density. In the petroleum era we have witnessed and continue to witness today, massive investments in pipelines, tankers and terminals, so that the set of exploitable petroleum resources now includes oil drilled in the world's most inhospitable Arctic climates and is collected from underwater wells literally miles deep. Naturally given the distances involved and the distribution of resources around the world, energy shipments now routinely cross political boundaries which has created a huge international trade in energy products where two centuries previous there was virtually none at all. Proposition 6 links this long chain of events to the changing power density of available energy sources.²

C.1.2 The Incentive to Upgrade

We start by providing a decomposition of power density into its component parts to understand how it may be subject to control. The decomposition links power density to directly observable and familiar characteristics such as crop yields, energy contents, and growth rates. We provide a similar decomposition for non-renewables in a subsequent section.

To start consider renewables that produce a physical harvest (timber, staple crops, fisheries, etc.) In these cases, we can write the steady state flow of energy harvested \mathcal{F} [Watts] from the renewable energy resource as the product of three things: the size of the resource stock S [kg]; its current growth rate r [1/time]; and the energy content of the yield, e

²Notice that this analysis can be expanded to the case of electricity generation and transmission. Increasing voltages is costly but reduces line losses, therefore there is an incentive to invest for as long as the power delivered is enough to compensate for the investment in higher voltage lines.

[Joules/kg]. $\mathcal{F} = reS$, where rS represents the physical harvest per unit time, and e translates this physical flow into an energy flow in units of Watts. The physical size of the stock S can in turn be written as the product of the physical density of the resource in the environment, δ [kg/m²] times the area containing the resource a [m²]. Making this substitution we obtain the flow of energy as $\mathcal{F} = (re\delta)a$. Power density is just the flow of energy per unit area or $\Delta = \mathcal{F}/a = re\delta$ [W/m²].³ Therefore, for a renewable resource that generates a continuous physical harvest flow - like a coppiced forest, biomass, etc. - its power density is proportional to its rate of growth, or recharge rate, r , its energy content e [Joules/kg], and its physical density in the environment, δ [kg/m²]. Since the harvest from the resource is rS we have $d = r\delta$ as the density of the harvest. Using this result we can now rewrite to find $\Delta = ed$, and $R^* = e/\mu g$. This leads to our next proposition.

Proposition C.2 *Quality versus Quantity. The extensive margin of energy collection is independent of the quantity of the resource available (measured by its physical density in the environment), but proportional to its quality (measured by its energy content).*

Proof. *In text.* ■

Proposition C.2 is quite intuitive. Recall the extensive margin is defined by zero net energy rent resources; that is, those for which transport costs completely dissipate the benefits of collection. Since transport costs are proportional to the amount collected, as is the energy contained within, it matters little whether we have an ounce of these marginal resources or a ton - marginal resources are marginal in whatever quantities we find them.

Once said, this result seems obvious, but it may explain much of the energy resource upgrading we see in the world today or in the past. Consider for example the age old collection of firewood and conversion to charcoal before transport. Charcoal has a higher energy content than wood, and therefore - via Proposition C.2 - will be collected at greater

³A 100kg forest growing at 10% per year generates 10 kg of firewood per year. Firewood contains 15 MJ per kg; and there are $31,536 \times 10^3$ seconds in a year. This piece of the forest provides 4.75 W on average for the year. If the physical density of the forest is such that it contains 50 kg of trees over each meter squared, then the power density of this forest resource is $(4.75/2)$.

distances. The fact that the conversion of fuel wood into charcoal is incredibly inefficient in a physical sense is irrelevant. Since stranded firewood resources have a zero opportunity cost, any degree of inefficiency is acceptable if a conversion to a higher e resource is possible.

Similarly today it is common to see energy resources upgraded to make transportation more efficient (lower μ), raise energy content (e), or both. For example, the upgrading of heavy oil not only raises its energy content and lowers its transport cost by lowering viscosity, it is also very energy intensive. While we may bemoan the energy and pollution consequences of this upgrading, the opportunity cost of using stranded resources is very low and hence the logic of doing so is impeccable. Compressing or liquifying natural gas is another example where the energy content (per unit volume) is raised, transportation made easier, and yet the process is quite costly.

To illustrate more concretely the incentives to upgrade resources, consider the following simple problem (along the lines of energy investments presented above). The examples discussed above in the text suggest that the role of upgrading is increasing in energy content e , while taking concentration d as given. With this in mind, power supplied to the core is given by $W^S(e, d) = \frac{1}{3} \frac{\pi e^3 d}{(\mu g)^2}$. Assume the costs of investing in resource upgrading to deliver energy content, e , is $k(e)$ such that $k'(e) > 0$ and $k''(e) > 0$.

Resource upgrading of this form is also common among energy resources without mass that generate electricity (solar, wind, hydro). To see why recall that in this case energy rents are driven to zero at the margin of exploitation, and this occurs at a distance given by

$$l^* = \frac{\Delta}{c} = \frac{\Delta}{I^2(\rho/a)}. \quad (\text{C.5})$$

the extensive margin now varies with power density, amperage and the details of the transmission network. Low amperage raises delivered power as do large lines and better materials. To understand how amperage matters, note that we have assumed any given location throws off power at rate Δ in Watts but said little about how this power is transmitted. In fact, this

power can be transmitted in a variety of ways because current times voltage equals Watts, or $W = VI$. Therefore, for a renewable energy source x with power density Δ we have a similar decomposition with $\Delta(x) = V(x)I(x)$. Substituting this into (C.5) we see that the extensive margin for source x is given by:

$$l^*(x) = \frac{V(x)}{I(x)(\rho/a)} \quad (\text{C.6})$$

Although voltage and amperage are not innate characteristics of energy sources, unlike energy contents and yields of renewables with mass, it is true that producing power from wind, solar, thermal generation or nuclear sources requires a set of generation technologies that have been optimized to meet their peculiar requirements. These generation technologies differ in the voltage produced. We can therefore think of otherwise homogenous providers of electricity as heterogenous in this regard, and from (C.6) we know the extensive margin relevant to any particular source x is rising in its voltage of transmission and falling in its amperage.

Putting these results together, we have a result much like we had before. Raising the power density of an energy source raises the flow of power produced but has no impact on the extensive margin if the voltage (quality) to amperage (quantity) ratio remain constant. And resources that would otherwise be stranded by their distance or low quality will either remain stranded or call forth endogenous investments to lower costs. While we prefer high voltage transmission (just as we prefer high energy content fuels) because it lowers line losses (or physical transport costs), altering voltage is of course costly just as improving roads, building canals, etc. to improve land transportation is costly. Indeed, our theory suggests that what appears to be excessively wasteful expenditures in energy used to ramp up voltages for the long distance transmission of renewables are but the mirror image of energy intensive methods to upgrade heavy oil or compress natural gas for non-renewables.

The optimization problem is given by $\max_e W^N = W^S(e, d) - k(e) = \frac{1}{3} \frac{\pi e^3 d}{(\mu g)^2} - k(e)$. The first order condition is given by $\frac{\pi e^2 d}{(\mu g)^2} = k'(e)$. Finally we can totally differentiate the first

order condition to obtain: $\frac{de}{dd} = \frac{e/d}{\epsilon'_{ke}-2} > 0$, where $\epsilon'_{ke} = ek''(e)/k'(e)$ and the sign follows from the assumption that costs are more convex than the benefits from upgrading resources.⁴

C.2 Patchy, Punctiform and Probabilistic Resource Distributions

In the main text we assume resources are uniformly distributed, the space containing resource plays is connected, and there is no uncertainty regarding whether the energy resources in question are present. These are strong assumptions, but for some resources they seem innocuous. For example, crops and woodlands typically satisfy these constraints at least over fairly large areas. But for other resources they fit less well and it is unclear how our analysis would change under these assumptions. For example, the available locations for resource exploitation may be patchy (containing holes) because of land use restrictions, habitat conservation, or noise considerations. The siting decisions for wind and solar farms certainly fit this description. In other cases, most notably fossil fuels, there are often a few very significant deposits surrounded by areas with little if any resource potential (the space contains resource “plays” with widely different power densities). We will refer to this case as one where the resource distribution is *punctiform*. In some other cases it is not clear ex ante whether resources are present in any specific location although there maybe a well defined probability distribution over them (oil and gas deposits come to mind). We refer to this case as one where the distribution of resources is *probabilistic*. We will show that often very little of substance changes with alternative resource distributions although the calculations become more lengthy and the expressions less transparent.

To understand why these complications rarely matter, recall our discussion of energy rents which allowed us to define the extensive margin R^* , for a resource of given power density Δ . Let this reliance of the extensive margin on the power density of resources be written as $R^*(\Delta)$. Then since all resources within this margin provide positive energy rents it

⁴Improving transportation or upgrading resources (by increasing energy content) are equivalent problems from the perspective of the energy producer. It also follows from the equivalence between electricity transmission lines and roads that increasing transmission voltage, therefore reducing Joule losses, is equivalent to reducing ρ which reduces land friction and transportation losses.

should be apparent that they will be exploited even if the resource base is not connected nor homogenous. And if we locate all such potential resources, identify their extensive margins, and then integrate over their relevant regions this (more complicated) sum of energy rents will equal the energy supply just as before. Apart from mathematical complications, patchy and punctiform resource distributions pose no special problem. Alternatively if we assume resources are present in specific locations with given probabilities, we can again identify $R^*(\Delta)$ and integrate over this space to find what would now be expected energy supply. And if the space defined by $R^*(\Delta)$ can be divided into many resource plays with identical and independent success distributions, then a law of large numbers result could be invoked to render expected energy supply equal to ex post energy supply. At bottom the reason why these complications do not matter much is the constant returns built into transport costs by the physics of the underlying problem. Moving an object twice as far is twice the work; moving an object with twice the mass is twice the work; and if movement is output and work (energy) is the input, this production function is CRS. The CRS feature of the problem allows us to aggregate easily, define boundaries simply, and replace patchy, punctiform and probabilistic resource distributions with much simpler connected and homogenous ones in many cases.

To see exactly how to incorporate complicated resource distributions, we construct two examples.

C.2.1 Patchy and Punctiform

It may be clear from the description above that the key complication is locating the various resources in space. To make the analysis tractable and transparent, we construct discrete resource distributions. Consider a division of the space surrounding the core into concentric circles that are then divided further into wedges created by extending rays from the core. The result, shown in Figure 1, is a sequence of land parcels we will refer to as resource plays.

Let there be $n = 1, \dots, N$ rays and $m = 1, \dots, M$ circles, then there are $N \times M$ resource

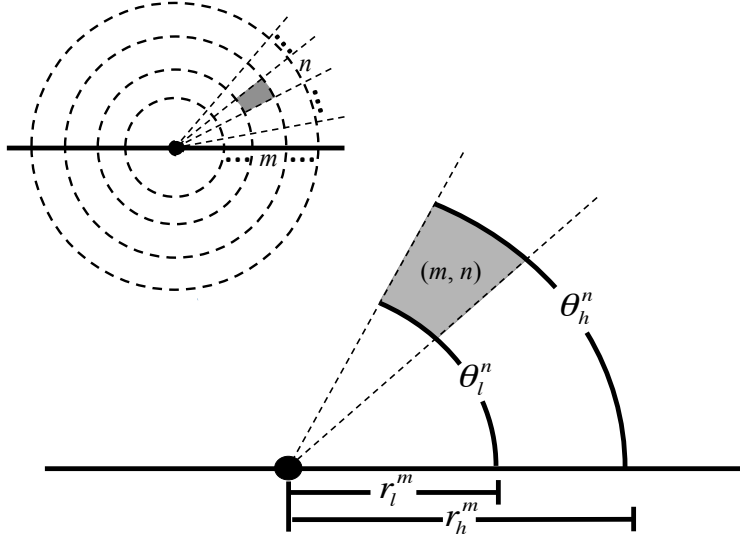


Figure 1: Uneven distribution of power density into parcels

plays each uniquely identified by the duple (m, n) . Suppose each play has an associated power density Δ_{mn} with geometrical shape characterized by its width $r^m = r_h^m - r_l^m$ and the angle of the wedge $\theta^n = \theta_h^n - \theta_l^n$. Where h and l refer to both the higher and lower radius bounds defining the play; and the higher and lower angles (measured in radians) that define its location in the plane. We can write the (maximum) energy supplied by any given resource play:

$$W_{mn} = \int_{\theta_l^n}^{\theta_h^n} \int_{r_l^m}^{r_h^m} v (\Delta_{mn} - cv) dv d\varphi$$

$$W_{mn} = \frac{1}{2} (\theta_h^n - \theta_l^n) ((r_h^m)^2 - (r_l^m)^2) \left(\Delta_{mn} - \frac{2}{3} c \frac{(r_h^m)^3 - (r_l^m)^3}{(r_h^m)^2 - (r_l^m)^2} \right) \quad (\text{C.7})$$

Since an energy supplier with play (m, n) supplies energy if the play provides positive energy rents, we need to account for this complication by noting that each density Δ_{mn} has an associated energy margin $\bar{R}_{mn} = \frac{\Delta_{mn}}{c}$. This implies the actual energy supplied to the

core by any resource play must be such that:

$$W_{mn}^S = \begin{cases} W_{mn} & \text{if } r_h^m \leq \bar{R}_{mn} \\ \bar{W}_{mn} & \text{if } r_l^m \leq \bar{R}_{mn} < r_h^m \\ 0 & \text{if } \bar{R}_{mn} < r_l^m < r_h^m \end{cases} \quad (\text{C.8})$$

where \bar{W}_{mn} has the same form as equation (C.7) but where r_h^m is replaced by \bar{R}_{mn} .

To find the aggregate energy supplied we add the n plays of each annuli m and then add all the annuli. Without further restrictions, the possibilities are very numerous. Therefore consider the case where each play within an annuli m has the same power density Δ_m . As well, order the power densities from lowest to highest such that $\Delta_0 = 0 < \Delta_1 < \dots < \Delta_m < \dots < \Delta_M$ so distant resources are the most power dense, and in order to eliminate potential gaps in our distribution we assume the width of each annuli is determined by the energy margins of the neighboring annuli. That is assume $r_l^m = \bar{R}_{m-1}$ and $r_h^m = \bar{R}_m$. Alternate assumptions are readily investigated. Using these assumptions we can now replace the definition of \bar{R}_m back in equation (C.7) to find the energy supplied to the core:

$$W_{mn} = \frac{1}{2} \frac{(\theta_h^n - \theta_l^n)}{c^2} \left(\frac{\Delta_m^3}{3} - \Delta_m \Delta_{m-1}^2 + \frac{2}{3} \Delta_{m-1}^3 \right)$$

Add over all wedges in the annuli m and over all the annuli M to find

$$W^S = \frac{\pi}{3c^2} \sum_{m=1}^M \Delta_m^3 \left(1 - 3 \frac{\Delta_{m-1}^2}{\Delta_m^2} + 2 \frac{\Delta_{m-1}^3}{\Delta_m^3} \right)$$

Two observations are in order. First, since the summation is over primitive determinants of the model, we could just as well replace this complicated sum with $\tilde{\Delta}$, where $\tilde{\Delta}$ is the power density of a hypothetical connected and uniformly distributed resource base yielding the same energy supply. $\tilde{\Delta} > 0$ by virtue of our ordering of power densities, and we can

write it simply as:

$$\tilde{\Delta} = \left[\sum_{m=1}^M \Delta_m^3 \left(1 - 3 \frac{\Delta_{m-1}^2}{\Delta_m^2} + 2 \frac{\Delta_{m-1}^3}{\Delta_m^3} \right) \right]^{1/3}$$

Second, if we alter the power density of our hypothetical resource base, $\tilde{\Delta}$, by $\lambda > 0$ this is equivalent to uniform scaling by λ of all power densities in the heterogenous resource zone. A moment's reflection will show that energy supply is homogenous of degree three in all power densities taken together. Therefore, for many purposes we can simply write

$$W^S = \frac{\pi \tilde{\Delta}^3}{3c^2}$$

and ignore the fact that the exploitation zone in question is both patchy and punctiform.

C.2.2 Probabilistic

Here we assume the power density of the resource is uniform across space and it is given by Δ_o . This implies all the resources found inside the margin of extraction given by $R_o = \Delta_o/c$ are going to be exploited. Divide this space as we did before using N rays and M circles to identify $N \times M$ resource plays but now assume each play has a probability q of having a resource with power density Δ_o in place and a probability $1 - q$ of being empty. Given Δ_o is uniform and constant, our previous assumptions imply the width of each annuli is equal to R_o/M . Thus, the geometrical shape of the parcel (m, n) is characterized by its boundaries set by $r_m^h = (m + 1) \frac{R_o}{M}$ and $r_m^l = m \frac{R_o}{M}$ and the angle of the wedge $\theta^n = \theta_h^n - \theta_l^n$. In the case where parcel (m, n) is not empty, we can calculate the same double integral we calculated for the case of patchy distributions and replace the values for r_m^l and r_m^h to find

$$W_{mn} = \frac{(\theta_h^n - \theta_l^n)}{2} \left(\frac{R_o}{M} \right)^2 \Delta_o ((m + 1)^2 - m^2) \left(1 - \frac{2}{3} \frac{1}{M} \frac{((m + 1)^3 - m^3)}{((m + 1)^2 - m^2)} \right) \quad (\text{C.9})$$

Replacing the definition for R_o and noting the wedges are of equal size given by $\theta^n = 2\pi/N$ we find:

$$W_{mn} = \frac{\pi}{NM^2} \frac{\Delta_o^3}{c^2} ((m+1)^2 - m^2) \left(1 - \frac{2}{3} \frac{1}{M} \frac{((m+1)^3 - m^3)}{((m+1)^2 - m^2)} \right) \quad (\text{C.10})$$

As we mentioned above, the power collected from parcel (m, n) is W_{mn} with probability q and it is zero with probability $1 - q$. Therefore, the expected value of energy provided by parcel (m, n) is:

$$E[W_{mn}] = q \cdot W_{mn} + (1 - q) \cdot 0 \quad (\text{C.11})$$

We can now aggregate across parcels and use the linearity of the expected value operator to find:

$$E[W^S] = q \cdot \bar{M} \frac{\pi \Delta_o^3}{3c^2} \quad (\text{C.12})$$

where \bar{M} is a constant given by:

$$\bar{M} = \sum_{m=0}^M \frac{((m+1)^2 - m^2)}{M^2} \left(3 - 2 \frac{1}{M} \frac{((m+1)^3 - m^3)}{((m+1)^2 - m^2)} \right) \quad (\text{C.13})$$

In this more complicated case very little seems to change. Power density again enters as a cubic as before since now the area of *exploration* rises with the square of the extensive margin and success is proportional to this exploration zone. As well, as mentioned earlier if the number of plays were large a variety of assumptions are available on the joint distribution across the plays that would render a law of large numbers result. The simplest case being the one employed above where each play is treated as an independent and identically distributed Bernoulli random variable.

C.3 Non-Renewables: Oil, Gas and Coal

Extending our framework to non-renewables presents several challenges. First, since using non-renewable energy today precludes you from using it tomorrow the exploitation zone must change over time as the resource stock is depleted. This is true because with non-renewables,

energy flows come from depleting the resource stock and not from harvesting the perpetual yield from a renewing resource. One simple and natural way to address depletion is to assume ongoing extractions hollow out the exploitation zone as the resource is extracted.⁵

Second, while we can for the most part ignore the potential impact current energy collection has on the future productivity of renewables (harvesting solar power today does not affect the likelihood of sunshine tomorrow), this is not possible with non-renewables. To see why, use the approach discussed above and assume all energy resources up to r have already been extracted. Then, the remaining non-renewable energy that could be supplied to the core is given by:

$$W^S = 2\pi \int_r^{R^*} (\Delta - cv) v dv = \frac{\pi\Delta^3}{3c^2} - \pi \left(\Delta r^2 - \frac{2}{3} cr^3 \right) \quad (\text{C.14})$$

where $R^* = \Delta/c$ as before. The intuition is clear. The economy loses the energy it would have been able to collect over the area already mined — this is, $\pi\Delta r^2$ —net of the energy it would have expended to bring this energy to the core, $(2/3)\pi cr^3$. Previous extractions raise the cost of current extractions, and the key economic problem is to determine the rate we wish to use these resources over time. To address this problem we will assume a time separable CRRA utility function that maps delivered energy into instantaneous utility flows, and maximize the discounted sum of these flows subject to resource availability and costs.

C.3.1 A Solow-Wan Reformulation

In order to solve our intertemporal energy supply problem we start by recognizing that our spatial model with reserves differentiated by location, can be rewritten as a standard problem where there is a fixed and given resource stock exploited subject to rising marginal

⁵There is a small literature examining least cost paths for depletion in situations with multiple deposits or resources. This literature, started by Herfindahl (1967), examines when, and under what conditions, a least cost order of extraction path will be optimal. Chakravorty and Krucic (1994) contains relevant references, some discussion, and a neat result showing the typical least cost path prediction does not hold up when the resources in question are not perfect substitutes in use. This possibility is ruled out in our one energy source set up, but would be relevant in any extension with two, less than perfectly substitutable, resource types.

extraction costs. A similar reformulation was first suggested by Solow and Wan (1976) in an environment where resources were differentiated by their grade, and it proves useful to do so here.⁶

To reformulate the problem along Solow-Wan lines, we first recognize that the exploitation zone has radius $R^* = \Delta/c$, and this exploitation zone implies a corresponding limit on recoverable reserves which we denote \bar{X} . These recoverable reserves are simply equal to $\bar{X} = \pi\Delta^3/c^2$ which again reflects our scaling law. But if the current resource frontier is $r(t) < R^*$, then the remaining recoverable reserves at t , which we denote $X(t)$ must be equal to

$$X(t) = 2\pi \int_{r(t)}^{R^*} \Delta \iota d\iota = \bar{X} - \Delta\pi r(t)^2 \quad (\text{C.15})$$

where $r(0) = 0$ since no resources have been extracted at the start of time. Cumulative extractions at t , are simply $\Delta\pi r(t)^2$.

Differentiating with respect to time, we find the needed link between remaining recoverable reserves and today's rate of extractions:

$$\dot{X} = -2\pi r(t)\Delta\dot{r}(t) = -W(t) \quad (\text{C.16})$$

The intuition is simple. As extraction proceeds, reserves are drawn down and the resource frontier expands. The frontier expands at rate $\dot{r}(t)$ as resources with power density Δ are reaped from a ring with density $2\pi r(t)$ per unit time. The last equality in (C.16) follows because the instantaneous change in the stock must equal $W(t)$ – the flow of energy extracted at t measured in Watts. This completes the first step of the reformulation.

The second step in the reformulation is to find the associated cost function for extractions. When $W(t)$ is extracted, it is divided between deliveries to the core $W^S(t)$ and energy used in transport which we denote $W^T(t)$. We refer to these costs as extraction costs. Because at any t there is a unique $r(t)$, $W^T(t)$ must equal $r(t)[c/\Delta]W(t)$ and hence $r(t)[c/\Delta]$ represents unit

⁶Solow and Wan (1976) suggested this reformulation in a short footnote; for a more illuminating treatment see section 2 of Swierzbinski and Mendelsohn (1989).

extraction costs. While this is useful, we need to eliminate $r(t)$ and purge the problem of all spatial elements. To do so use (C.15) to substitute for $r(t)$ as a function of remaining reserves $X(t)$ and total reserves \bar{X} . With some simplification, we can now write the relationship between energy supplied to the core, $W^S(t)$, current extractions, $W(t)$, remaining reserves $X(t)$, and recoverable reserves \bar{X} , as:

$$W^S(t) = [1 - C(X(t))]W(t) \quad (\text{C.17})$$

$$C(X) = \left(1 - \frac{X}{\bar{X}}\right)^{1/2} \quad (\text{C.18})$$

where we now interpret $C(X(t))W(t)$ as the cost of extracting $W(t)$ units of energy from a homogenous pool of recoverable reserves \bar{X} , when remaining reserves equal X . $C(X(t))$ is therefore the unit extraction cost function (where we have suppressed its reliance on recoverable reserves, \bar{X}). With this machinery in place, we solve for the optimal extraction path.

A social planner maximizes the welfare of a representative consumer who values the energy services available for consumption in the core. By choosing service units appropriately, utility is defined over net energy supplied. The planner has a CRRA instantaneous utility function with coefficient of relative risk aversion equal to $\sigma > 0$. Social welfare is:

$$\max_{W(t)} \int_0^\infty e^{-\rho t} U(W^S(t)) dt \quad \text{where} \quad U(W^S) = \frac{(W^S)^{1-\sigma} - 1}{1-\sigma} \quad (\text{C.19})$$

The planner maximizes (C.19) subject to the constraints (C.16) and (C.17). We write the current value Hamiltonian as

$$\mathcal{H} = U[(1 - C(X(t)))W(t)] - \lambda(t)W(t) \quad (\text{C.20})$$

where $\lambda(t)$ is the co-state variable associated with the stock of resources. The optimality

conditions are given by:

$$\frac{\partial \mathcal{H}}{\partial W(t)} = U'(W^S(t))(1 - C(X(t))) - \lambda(t) = 0 \quad (\text{C.21})$$

$$\frac{\partial \mathcal{H}}{\partial X(t)} = -U'(W^S(t))C'(X)W = \rho\lambda(t) - \dot{\lambda}(t) \quad (\text{C.22})$$

with transversality condition $\lim_{t \rightarrow \infty} e^{\rho t} \lambda(t) X(t) = 0$.

Using the definition of the utility function in (C.19) and taking the time derivative of (C.21), substituting in (C.22), and rearranging we find one differential equation linking the current rate of extractions to cumulative extractions:

$$\frac{\dot{W}(t)}{W(t)} = -\frac{\rho}{\sigma} - \frac{C'(X)}{1 - C(X)} W(t) \quad (\text{C.23})$$

A second differential equation is provided by (C.16) while one initial condition and the transversality condition close the system.

The behavior of the dynamic system is presented in Figure 2. The $\dot{W}(t) = 0$ isocline is depicted by the solid curve in Figure 2. This curve is positive for values of $X(t) \in [0, \bar{X}]$ and has a maximum value when cumulative extraction is one quarter of total reserves; i.e. with remaining reserves $X(t) = 3\bar{X}/4$ and cumulative extraction $\bar{X} - X(t) = \bar{X}/4$. The $\dot{X}(t) = 0$ isocline is coincident with the horizontal axis. At all points above the $\dot{X}(t) = 0$ isocline, movement must be rightwards to extract all reserves, giving arrows of motion in the positive direction parallel to the horizontal axis. At points above the $\dot{W}(t) = 0$ isocline, extractions must be increasing since costs are currently too low; below the isocline just the opposite is true. This information is captured by the arrows of motion shown.

Assume we start with a new resource and hence cumulative extractions are zero. Since the arrows of motion near the origin imply all movement must be upwards and to the right, we know the system must move immediately to an initial extraction point like that shown by $W(0) = W_0$. From this initial point, the arrows of motion indicate we move upwards

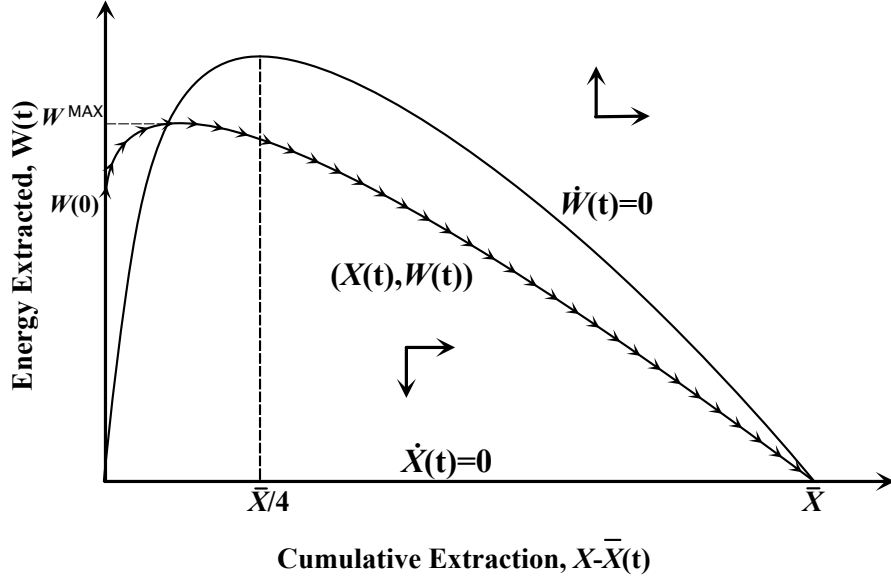


Figure 2: Optimal Extraction Rule

and to the right and cut the $\dot{W}(t) = 0$ isocline at zero slope. Once we cross this isocline, the arrows of motion tell us the extraction path must turn downwards and the transversality condition requires the path slowly approach \bar{X} on the horizontal axis. Working backwards it is now apparent the transversality condition chooses the initial W_0 and this choice has to feature less extraction than that given by the peak of the $\dot{W}(t)$ isocline.

Somewhat surprisingly, extractions must at first boom and then (optimally) bust. This is true despite the uniform distribution of resources across space; despite the very conventional form of intertemporal utility and absence of demand shocks; and despite the absence of learning by doing, technological change or exploration activity. Moreover, the peak in extractions is greater the more power dense the underlying resource.

At bottom the cause is the scaling law linked to the spatial structure of the model but to understand why this is true, we need to understand the two quite different motivations governing optimal depletion. First, and not surprisingly, there are the standard Hotelling motives arising from the finiteness of the resource stock and the impatience of our planner. For example, if the costs of bringing energy to the core was zero (but there remained a finite resource stock available for use), then the shadow value of the resource *in situ* rises at the

rate of time preference. Energy extracted would equal energy supplied to the core and the time profile for extractions would be given by

$$\frac{\dot{W}(t)}{W(t)} = -\frac{\rho}{\sigma} < 0 \quad (\text{C.24})$$

where ρ is the discount rate, and σ the elasticity of marginal utility from the CRRA specification. Since optimality requires the value of marginal utilities discounted to time zero to be equalized across all periods, this is achieved by energy consumption falling at a rate proportional to time preference and the elasticity of marginal utility. This motivation follows from the finiteness of the reserves; it predicts a declining path for extractions; and, it reflects the forces identified in Hotelling's classic work (Hotelling 1931).

Second, and less familiar, are what we could call Ricardian motives. These motives come from the fact that reserves differ in their Ricardian rents: energy resources very proximate to the core have large rents and are very scarce; while very distant ones have very little rent but are abundant. Once we translate this feature of our spatial structure - via the Solow-Wan reformulation - into an implication on extraction costs, it implies that differences in Ricardian rent across reserves are now reflected in extraction costs that rise rapidly with cumulative extraction. Since any unit extracted today raises the cost of all future extractions, all else equal, it pays to shift these extraction costs into later periods. These Ricardian motives argue for a delay in extractions or what is the same, a rising path of extractions over time. Ignoring the Hotelling term given above, an extraction path that reflects only Ricardian considerations is given by

$$\frac{\dot{W}(t)}{W(t)} = -W(t) \frac{C'(X)}{1 - C(X)} > 0 \quad (\text{C.25})$$

Equation (C.23) shows optimal extractions is the simple sum of the right hand sides of (C.24) and (C.25), and hence the interplay of these two forces produce a boom and bust path for energy production.

Descriptively, the result follows because the Ricardian motivations initially dominate

Hotelling considerations. Analytically, it follows because at the very first instant of time, energy consumption must be positive $W^S(0) > 0$, and $C'(X(0)) = -\infty$ implying $\dot{W}(t) > 0$ at least initially. And as extraction proceeds $W^S(t)$ must approach zero (the resource is finite) and $C'(X(t))$ increases. Therefore, the Ricardian forces fall over time and are eventually dominated by the Hotelling ones.

More deeply, the impact of using up the very first unit of resources on subsequent extraction costs is so costly, $C'(X(0)) = -\infty$, not because these initial energy resources have the greatest rents (which they do) but because they are so scarce in relation to the resource pool whose extraction costs are now raised. Scarcity drives the result and high rent resources are so scarce because of our scaling law. To see why, recall that energy rents fall linearly with distance, but the quantity of reserves rises with the square of distance. This implies low rent resources are abundant, and high rent resources are scarce. Increasing the power density of the energy source raises rents everywhere, but also brings in play new low rent resources at the margin of exploitation. Consequently, the motivation for pushing extractions into the future is strengthened and the peak of extractions rises.

While it is well known that the typical Hotelling's prediction can be overturned in a variety of settings, the boom and bust in extractions is a necessity in our framework and not a possibility.⁷ Moreover, it follows from our scaling law which has the dual cost implications reflected in (C.18). The spatial setting provides us with a neat analytical representation for a well known empirical fact: high rent resources are scarce and low rent resources abundant - and then suggests that the logical implication of this fact is that both Ricardian and Hotelling motives drive optimal extractions. Ignoring spatial elements and their attendant impact on rents removes a key force driving optimal extractions; and taking them into account suggests an interesting parallel. Non-renewable resource extraction should be concentrated or bunched in time, just as renewables energy extractions should be bunched across space. We summarize this result in the following proposition.

⁷The first observation that extractions may boom and bust is often credited to Livernois and Uhler (1987).

Proposition C.3 *Assume intertemporal utility is of the CRRA form, then the optimal depletion path has extractions rising to a peak and then declining. Peak extractions are rising in the power density of the energy resource.*

Proof. *In text.* ■

The proof of the second part of Proposition C.3 is as follows. For any $Z = \bar{X} - X$ constant, W^{MAX} in Figure 2 is increasing in \bar{X} . Setting $dW/dX = 0$ we find $\frac{\partial W}{\partial X} = 2\frac{\rho}{\sigma}Z\frac{(\bar{X}^{1/2}-Z^{1/2})-\frac{1}{2}\bar{X}^{1/2}}{(\bar{X}^{1/2}-Z^{1/2})^2}$. Therefore, $\frac{\partial W}{\partial X} > 0$ if $\frac{\bar{X}}{4} > (\bar{X} - X)$ which always holds as the peak in the extraction always occurs to the left of the peak in the $\dot{W} = 0$ locus.

D Measuring Power Density

We start by developing the theory required for measurement. While there exist in the literature estimates of power densities for many energy sources, how these figures are constructed is unclear and rarely documented adequately. Measuring power density for some renewable resources is fairly straightforward; for example, crops dedicated to biofuels or human consumption can be turned into energy equivalents and then power flows by taking account of crop cycle, length, and area planted; similarly coppiced forests can provide stable flows of wood products for heating and cooking needs and similarly occupy well defined areas. In these cases, the renewable flows are captured by the physical quantity of fuel reaped from a resource stock.

In other cases the renewable flow does not have mass but provides either kinetic or electromagnetic energy we capture and exploit directly. In these cases the measurement is straightforward and represents the potential of these flows. For example, the power density of solar is easily estimated once we are armed with knowledge of insolation potential at a location. Similarly, wind or wave farms provide useful kinetic energy and we again find the potential power flow from the resource per unit area.

In the case of non-renewables, measurement is generally more challenging. One common

method is to calculate the actual physical footprint of an energy facility's size and divide this by the current energy output. So for example, if a coal based generating station produces a constant flow of 1MW, and the mine and generating station takes up 1 km², then its power density is simply $[1 \times 10^6 \text{W}]/[1 \times 10^3 \text{m}]^2 = 1 \text{W/m}^2$. There are several obvious problems with this method. First, the measure of power density is technology dependent. Improvements in generation technology will affect power density, and therefore power density will not be a characteristic of an energy source but rather reflect current technology in place. Second, it is difficult to know which "inputs" we should include in the measurement. For example, by including mining, crushing, and generating facilities in the calculation we make implicit decisions about which facilities to include and which to exclude. Should we also include the area taken up by transmission lines, relay stations, and other parts of the grid? In the case of oil, do we include pipelines, refineries and gas stations? If pipelines are buried and transmission lines are above ground, how do we deal with this?

How these decisions are made will materially affect the calculation. Replication of any measure produced will be almost impossible. As a result any comparison across measurement attempts will be far to reliant on the individual judgment of the researcher.

In order to resolve these issues, we present a method to measure power density for both renewables and non-renewables that is independent of technology, easy to replicate, and allows for a comparison of power densities across energy types. To do so we use restrictions from economic theory to help aggregate resources along the dimensions on which they differ, and we provide measures assuming an ideal environment where resource stocks are homogeneous and where the only costs of exploitation are those arising from energy costs. Our goal is to develop measures that reflect only the physical properties of the resource and not our current, past, or future ability to reap these energy flows.

We start our discussion with the case of renewables since it is simpler to understand and relatively straightforward in practice.

D.1 Power Density for Renewables

D.1.1 Theory

Any renewable flow of energy resources produced from a renewable resource \mathcal{F} [Watts] can be written as the product of the current (and steady state) physical size of the resource stock S [kg] multiplied by the energy content of the resource, e [Joules/kg] and a growth rate r [1/time]. This implies the flow is given by $\mathcal{F} = reS$.⁸ The physical size of the stock can be similarly written as the product of the physical density of the resource, d [kg/m²] times the area actually used by the resource a [m²]. Making this substitution we obtain the flow of energy as $\mathcal{F} = (red)a$. Power density is just the flow of energy per unit area or $\Delta = \mathcal{F}/a = red$ [W/m²].

Power density is proportional to the product of the maximal rate of regeneration, r , which measures the percentage rate of growth of the resource in an unconstrained environment. Perhaps not surprisingly, a renewable energy source that grows twice as fast has twice the power density. It is also proportional to the energy content of the fuel, e , measured in [Joules/kg], again perhaps it is not surprising that energy density matters but the specific form is of course not obvious. Finally power density also depends on a fuel's physical density, d , measured in [kg/m²]. All else equal a fuel that produces a greater output in terms of harvest weight gives more energy.

Two special cases deserve attention. The first case applies to resources like wind or solar where there is no associated physical product. In this case we replace the stationary harvest of the resource that we used above by a measure of an average flow per unit time, and then apply energy equivalents to obtain a measure in terms of energy production per unit time. For example, average wind flow per unit area in a given location can be transformed into its kinetic energy equivalent per unit time; average solar insolation in a location is already measured in Watts per unit area terms, and measures of the energy in wave motion

⁸For example, a 100 kg forest growing at 10% per year generates 10 kg of firewood per year. Firewood contains 15 MJ per kg; and there are $31,536 \times 10^3$ seconds in a year. The forest provides 4.75 W on average for the year.

can likewise be measured in power density terms. In these three cases while there is no physical resource reaped, power density is simply measured by the potential energy flow these resources deliver per unit area, per unit time.

The second case arises when the harvest from a resource affects the resource stock size, and in turn the power density of the energy source. For example this is most likely to occur when resources are crops, forest land, or hydro power. In these situations it is natural to pin down the power density of the resource by assuming the energy supplier manages the resource to maximize the discounted flow of energy over time.

To investigate further it is necessary to be specific about the dynamics of resource growth. Suppose the energy source is a renewable fuel with natural growth given by $G(S)$ with $G(0) = G(K) = 0$. As is standard let G be strictly concave and let S denote the stock in physical units. For example, the energy source could be a forest, an area dedicated to biofuels, or even an area dedicated to solar or wind power. It is helpful to first take a specific example with explicit units. To that end, let $G(S) = rS[1 - S/K]$ and the maximum sustainable yield harvest, H^{msy} , as one possible plan for taking from the resource. Then in perpetuity this harvest is given by:

$$H^{msy} = G(S^{msy}) = G(K/2) = rK/4 \tag{D.1}$$

where r is the intrinsic resource growth rate and K is the carrying capacity (i.e. a growth rate times a stock level). Now consider units explicitly. If K is measured in kilograms and $K = 100$ kg, and $r = 10\%$ per unit time, then the sustainable harvest is $(.1)(100)/(4)=2.5\text{kg/unit time}$. If we multiply this quantity by the energy content of the fuel in [Joules/kg] denoted by e , we obtain a measure of Joules per unit time that could be harvested from the resource. Choosing to measure time in seconds, we obtain *Watts*. The final step is to divide this flow of power by the area of exploitation needed to maintain it. Since the carrying capacity is K kg, and if the fuel has a physical density, d , measured in [kg/m²], then the total area needed

for this resource flow is K/d . All this implies we can write power density for this renewable energy resource, Δ , as:

$$\begin{aligned}
 H^{msy}e &= \mathcal{F}[\text{Joules/second}] = [\text{Watts}] \\
 \Delta &= \frac{(rK/4)e}{K/d} = \left[\frac{\text{Watts}}{\text{m}^2} \right] \\
 \Delta &= \gamma red \quad \gamma > 0.
 \end{aligned}
 \tag{D.2}$$

Power density is the simple product of three fundamental, commonly used, and potentially observable characteristics of an energy source, plus one behavioral component captured in the parameter γ . The remaining term in power density is the factor of proportionality γ which captures the intensity of harvesting. To see this note that if harvesting results in a steady state stock equal to a fraction of the carrying capacity given by κK , then $\gamma = \kappa(1 - \kappa) < 1$. Harvesting zero implies $\kappa = 1$, $\gamma = 0$ and $\Delta = 0$; harvesting sufficiently high to cause extinction implies $\kappa = 0$, $\gamma = 0$ and again $\Delta = 0$. The example given above has $\kappa = 1/2$ and $\gamma = 1/4$. We chose this example for a particular reason: if the resource owner was interested in maximizing total energy collected over an indefinite future, then the owner would adjust their take to match that of the maximum sustainable yield. This is obvious, but more generally, if the owner discounts the value of future versus current energy flows, the optimal stationary harvest maximizing this objective would lead to $\delta = G'(S^*)$ where δ is the discount rate on future periods. This is an application of a well known result in resource economics.⁹

D.1.2 Empirics

Renewable energy has two final uses: food and fuel. One use is supplying energy to maintain bodily functions, while the other use is supplying energy for heating, light and power

⁹ The optimal stock and attendant harvesting is set only by impatience and is independent of prices. In our simple example with logistic growth, power density is simply $\Delta = \gamma red$, and $\gamma = [1 - \delta/r]^2/4$ is positive as long as suppliers discount rate is not too high.

Table 1: Power Density Crops

Crop	Yield [kg/m^2]	Energy Content [MJ/kg]	Power Density W/m^2
Sugar cane	7.16	17.01	3.86
Sugar beet	4.87	9.75	1.50
Cassava	1.25	6.69	0.27
Bananas	2.05	3.73	0.24
Rice, paddy	0.43	15.5	0.21
Potatoes	1.78	3.22	0.18
Sweet Potatoes	1.25	3.60	0.14
Wheat	0.30	15.07	0.14
Barley	0.26	14.74	0.12

applications. In Table 1, we present figures on the power density of various staple crops from around the world. The figures presented for yields are estimates of “typical” yields for these crops in a system with sustainable rotation (fallow periods). As shown food crops, even staples, offer relatively small power density. Even the powerful potato offers only 0.18 W/m^2 in terms of food for fuel, but some tropical crops such as cassava (.26 W/m^2) and bananas (.24 W/m^2) provide much more. Crops that have found use as biofuels such as sugar cane (3.87 W/m^2) and sugar beet (1.50 W/m^2) have power densities one order of magnitude larger than other crops.

Table 2 presents figures on the power density of forests for six regions of the U.S. Since the productivity of forests and their composition varies so too does their power density. For example, the South Central forests have the highest growth rates (column three) whereas the North Central forests have the greatest percent of Hardwoods (column five). The power density figures are again relatively small and on the order of 0.1 to 0.15 W/m^2 .

Comparing Table 1 to Table 2 we see that in general wood provides lower power density than crops. This is true even though wood density, d , and energy content, e , are much higher than the standard crop. The result follows, because a forest’s rate of growth, r , is very low relative to that provided by annual crops.

Finally in Table 3 we present estimates for solar and wind energy for six regions in the

Table 2: Power Density of Wood

Forest Region	Total Acres (1,000s)	Ave. Prod. (ft ³ /acre/yr)	Percent Softwood	Percent Hardwood	Average Power Dens. (W/m ²)
Northeast	79,803	57.10	25.21%	74.79%	0.10
North Central	84,215	66.54	18.72%	81.28%	0.12
Southeast	85,665	80.22	41.00%	59.00%	0.14
South Central	118,364	84.69	35.20%	64.80%	0.15
Rocky Mountain	70,969	52.00	90.29%	9.71%	0.08
Pacific Coast	75,197	81.71	89.01%	10.99%	0.12

Table 3: Power Density of Wind and Sun

Region	Solar	Wind	
	Power Density [W/m ²]	Speed [m/s]	Power Density [W/m ²]
Northeast	164	7	122
North Central	226	9	281
Southeast	217	7.5	136
South Central	219	7	115
Rocky Mountain	256	9.5	296
Pacific Coast	229	7.5	144

US. In all cases, we measure the potential provided by the resource rather than measures of our current ability to reap the resources in question. The power density of solar energy captures the yearly average amount of radiation collected by one squared meter of surface with tilt equal to the latitude of the point of measurement. Solar radiation data is reported in the first column on Table 3.

The amount of power that can be extracted from wind is a cubic function of the speed of the wind (column 2), and it is proportional to the area of the cross-section perpendicular to the velocity of the wind. Assuming one squared meter cross-section gives us the amount of power extracted at any given velocity. If we further assume one meter squared of land is used by one meter squared of cross-section then we can find the power density of wind. This is what we show in the last column of Table 3.

It is interesting to note the huge differences across these tables measuring the power density of crops or timber versus the raw energy flows in solar. Photosynthesis is, even

in the best environments, a very inefficient process taking solar power from the sun and then via nature's capital equipment turning it into biomass. Estimates on this efficiency vary but a common estimate is below 1% efficiency, and this is similar to what a very naive comparison of these tables might suggest.

D.2 Power Density for Non-renewables

D.2.1 Theory

The method we presented earlier to calculate the power density of renewables cannot be applied directly because non-renewables are obviously very different. For example, the flow of energy obtained from non-renewables is proportional to the *change* in the resource stock over time, and non-renewables are not spread over vast areas but instead found in subsurface deposits of considerable thickness. To proceed we deal with these issues in turn. First, assume the resource *was* distributed uniformly on the surface of a two dimensional plane. Then the change in the resource stock would equal the physical flow of the resource, which we will denote by ϕ [kg/m²·s], times the area exploited, a [m²]. This gives us the kilograms mined per second. Multiplying by energy content implies the power density of a non-renewable resource distributed on the surface, could be written as:

$$W = e\phi a \tag{D.3}$$

$$\Delta = W/a = e\phi \tag{D.4}$$

This makes perfect sense: the flow of energy comes from the change in a stock; the magnitude of the resulting energy flow is determined by the stock's energy content; the change in the stock is equal to the product of the area extracted times the physical flow of the resource mined; and dividing by area exploited, we obtain the result. For future purposes it is also useful to note that the zero rent margin for this non-renewable is again equal to $R^* = e/\mu g$ since ϕ kg of the ore is being transported for each square meter mined per second; and hence

$c = \mu g \phi$. Therefore, we have a quality/quantity result similar to Proposition 2.

Unfortunately, this measure of power density only holds in situations where non-renewables are distributed very thinly across Earth's surface. For example, shallow deposits of the non-renewable peat may well fit this description. In the vast majority of cases however non-renewables are not thinly distributed as surface deposits. To tackle this problem we need to develop an appropriate measure of available resources in subsurface deposits. Energy resources will be brought to the surface and then transported to the core only if they provide positive energy rents; and this implies there is now an extensive margin of exploitation for any mine in terms of depth.

Consider a hypothetical resource owner with resource rights to one meter squared of surface area in our exploitation zone of homogenous resource quality. If this resource owner extracts a 1m^3 cube of energy resources with energy content $e[\text{J/kg}]$ and volumetric density $d_v[\text{kg/m}^3]$, then this cubic meter has mass of $d_v[\text{kg}]$ and weight of $d_v g[\text{N}]$. The total energy contained in this cube would be simply $ed_v[\text{J}]$ and if it was exhausted in one second the power delivered would be ed_v [Watts].

Now consider resources contained beneath this 1m^2 . If resources are located at distance $\eta[\text{m}]$ from the surface, then the work needed to bring them to the surface would be just $gd_v\eta[\text{J}]$ since work must be done to offset gravity. Energy rents in this case are just $ed_v - gd_v\eta$. Resources where the energy cost of extraction equals their entire energy content produce zero energy rents and are located at depth $\eta^* = e/g$. The net energy available must account for the energy costs of extraction and is found by the following simple integration.

$$\Delta = \int_0^{\eta^*} [ed_v - gd_v\eta] d\eta = \int_0^{\eta^*} ed_v [1 - (g/e)\eta] d\eta = e^2 d_v / 2g \quad (\text{D.5})$$

where e and g are as defined before, while d_v is the volumetric density of the resource in $[\text{kg}/\text{m}^3]$.

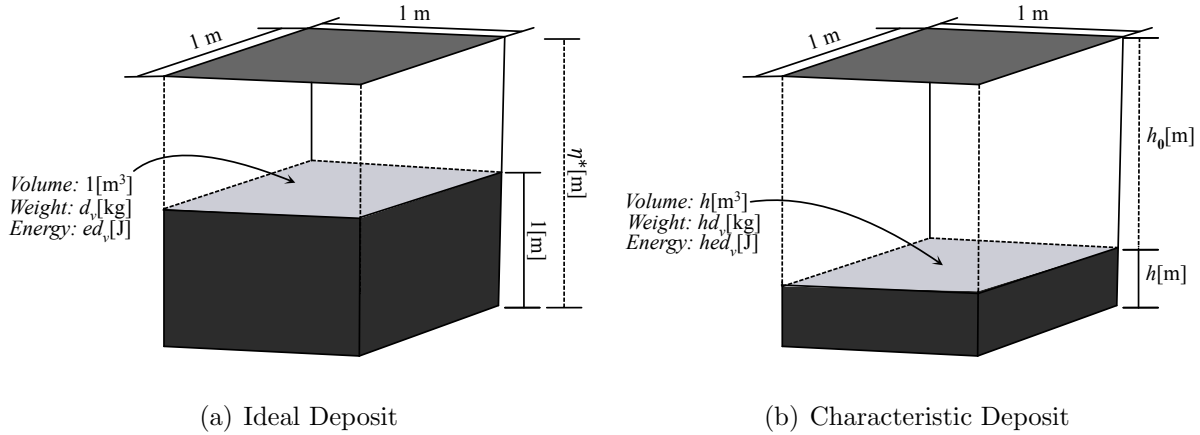


Figure 3: Non-renewable deposit

D.2.2 Empirics

Non-renewable resources are found in all continents. Reservoirs can be shallow or deep; they can be large or small; and fuels can be of different qualities. For example, the “quality” of coal as measured by its energy content depends on the depth of burial. Lignite coal is the coal with the lowest energy content and it is formed when Peat is buried between 200 and 1500 meters during the coalification process. The process of Lignite coalification increases the energy content from around 13 MJ/kg for Peat to 16 MJ/kg but it almost doubles its volumetric density from around 355 kg/m³ to 700 kg/m³. The formation of Bituminous coal occurs at greater depths (between 2500 and 6000 meters), and in the process the energy content increases to 32 MJ/kg and the volumetric density again increases to over 900 kg/m³. Anthracite coal is formed deeper than other ranks of coal (between 6000 and 7500 meters). It is the purest form of coal (up to 96% pure carbon) with an energy content of 35 MJ/kg and volumetric density as high as 950 kg/m³. Erosion, earthquakes and volcanic activity can expose these deposits or bury them even more. For example, the maximum depth of deposits in Argentina is 600 meters for sub-bituminous coal and minimum seam thickness of 1.8 meters, Ukraine has deposits of bituminous coal that are 1600 meters deep and have a minimum seam thickness of 0.5 meters. Australia has Lignite coal deposits at a maximum depth of 300 meters and minimum seam thickness of 3 meters. Many places in the world

also feature surface deposits.

Oil and Natural Gas deposits can also be found at various depths and the quality of the resources also depends on the depth of burial, although the process of formation is quite different. The formation of petroleum occurs anywhere between 2500m and 4500m. The temperature and pressure at this depth combine to transform decayed organic matter into oil. Just as with coal, erosion and other geological forces can bury the oil deposit further or expose it. Oil deposits can be right at the surface of Earth's crust or they can be as deep as 12,000 meters; extreme temperatures below this point are likely to bake most of the crude in the deposit.

These complications mean that our theoretical construct of a continuous ideal deposit running η meters in depth from the surface is rarely obtained. To amend our calculation for any specific deposit, suppose the upper limit of the deposit is located at a distance h_0 from the surface, and suppose the thickness of the deposit is h , so the maximum depth of the deposit is $h_0 + h$; as shown in Figure 3.(b). Because the resource rights are over one meter squared the dimensions of the deposit are simply $h[m^3]$. In this case to measure the power density of this deposit, we use:

$$\Delta = \int_{h_0}^{\min\{h_0+h,\eta^*\}} [ed_v - gd_v\eta] d\eta = \int_{h_0}^{\min\{h_0+h,\eta^*\}} ed_v [1 - (g/e)\eta] d\eta$$

$$\Delta = \min\{h [d_v e - gd_v(h_0 + h/2)], e^2 d_v / 2g - h_0 (ed_v - (gd_v h_0) / 2)\} \quad (D.6)$$

We now use this formula to calculate the power density for different types of coal in Table 4 below. To give an idea of the resulting power densities we present a set of alternative estimates. We present estimates assuming a deposit's thickness, h , is either 1 or 10 meters; estimates for deposit depths, h_0 , of 0 and 10,000 meters; and since coal varies in quality due to the influence of time and pressure we present estimates for a precursor (peat), low quality brown lignite coal; and higher quality bituminous coal.

As shown, higher quality coal has both higher energy content and greater volumetric

Table 4: Power Density of Different Coals

Resource	Energy Content [MJ/kg]	Volumetric Density [kg/m ³]	Power Density [GW/m ²]			
			$h_0=0$ m		$h_0=10000$ m	
			$h=1$ m	$h=10$ m	$h=1$ m	$h=10$ m
Anthracite	33	1400	46.2	462	46.2	462
Bituminous	33	913	30.1	301	30	300
Lignite	16	865	13.8	138	13.8	138
Peat	14	310	4.34	43.4	4.31	43.1

density. Time and pressure increase both and this implies given our formula that higher quality coal is significantly more power dense than its precursor peat. We find the deposit depth has little effect on the energy contained in the mine, and hence comparing the power density of resources at zero depth versus those at 10,000 meters reveals only small differences. And finally, the power densities are very large. To understand the scale of the energy shock created by a movement to coal we compare these figures with similar figures for wood in Table 2.

Several observations are in order. First, a naive comparison of the figures in these tables reveals large differences in power densities which naturally suggests the introduction of coal would be a massive energy shock to an economy where wood provided most thermal energy. While we believe that this was indeed the case, a comparison of these figures requires careful attention. The power densities measured for renewables represents the flow of power we capture on our one square meter of space *ad infinitum*. The wind blows, the sun shines, crops grow and forests mature providing a potential, and constant in steady state, flow of energy we can reap. In the case of non-renewables, the energy flow depends on how fast we extract the resources contained in one square meter. A straight ahead comparison of power densities across these tables implicitly assumes we are consuming all of the non-renewable energy resources from one square meter (and all of its subsurface deposits) per second. This is incredibly fast. A better interpretation of the differences in power densities across these tables is that the potential for reaping power from non-renewables is just vastly greater.

A second observation is that any existing low cost transportation option has a bigger

impact on energy supplies when the resource is more power dense. This is an implication of the magnification effect shown in Proposition 2. Therefore, existing rivers and roads would magnify the impact of coal far more than they magnified the impact of biomass resources.

Third, any comparison of wood and coal using just energy contents will severely underestimate the impact of a move to coal. Coal offers perhaps 25 MJ/kg while wood 15 MJ/kg, which means we would be willing to transport coal further than wood given similar transport technologies, but not much further. But comparisons like these miss an important point captured in the complementarity effect of Proposition C.1. Because coal is so much more power dense, the incentive for expanding the existing exploitation zone is much larger. And it was these endogenous improvements (canals, railroads, ports) in response to coal that, combined with its power density, created a huge and long lasting energy shock.

D.3 Data used in the Tables.

D.3.1 Table 1: Crops.

Data on 2011 global crop yields in Table 1 is provided by the Food and Agriculture Organization of the United Nations' (FAO) FAOSTAT Production database (2013). Global crop yield is calculated automatically by the FAO database by dividing annual crop production by area harvested. Production and harvest figures are reported to the FAO by individual countries via questionnaire or national agriculture publications. Detailed information on the collection methods for this data can be found in the Metadata section of the FAO Statistical Yearbook (2012, 357-358) as well as in the entry for agricultural production on the FAO's Methods & Standards Webpage.

Data on energy content for the crops given in Table 1 is provided by the United States Department of Agriculture, Agricultural Research Service's National Nutrient Database, Release 25. Energy content is given in kilocalories (kcal) per 100 grams, which is converted to MJ/kg by multiplying by 4.1868 kJ/kcal and then dividing by 100 to convert kJ/100g into MJ/kg.

D.3.2 Table 2: Forests.

Average power density for a forest in each of the United States regions listed in Table 2 is calculated by the formula:

$$\begin{aligned} & (\text{Average Productivity in Region (hg/m}^2\text{/year)} \times \text{Power Density of Average Hardwood} \\ & (\text{J/m}^2\text{/year)} \times \text{Percentage Hardwood in Average Forest}) + (\text{Average Productivity (kg/m}^2\text{/year)} \\ & \times \text{Power Density of Average Softwood} \times \text{Percentage Softwood in Average Forest}) \end{aligned}$$

The power density of average hardwoods and softwoods is calculated from data provided by Engineering Toolbox. To be precise, power density for individual tree species is calculated by dividing the recoverable heat value of a dry cord of wood (million BTU/cord) by the weight of a dry cord (lb/cord) given by Engineering Toolbox to get the recoverable heat value per pound of wood (million BTU/lb), which is then converted to recoverable heat value per kilogram of wood (MJ/kg), also called potential heat value per kilogram of wood, using conversion factors of 1 lb = 0.4536 kg and 1 million BTU = 1055.06 MJ. Calculations in Table 2 are based on an average dry hardwood density of 35.52 lb/ft³ with a 14.89 MJ/Kg potential heat value, and an average dry softwood density of 27.45lb/ft³ with a 14.87 MJ/Kg potential heat value.

Average hardwood power density is then calculated by averaging the potential heat value per kilogram of wood for five species (aspen, cottonwood, red oak, red maple and white oak) considered common in American forests by the USDA (2007, 62). An identical calculation is done for four species of softwood (hemlock, ponderosa pine, balsam fir and white pine) also considered common in American forests by the USDA (2007, 62) to get an average softwood power density. It should be noted that the variation in potential heat value per kilogram between different species of tree is always less than 1%, so average hardwood and softwood heat values do not vary much if different species of tree are chosen for the calculation.

Data on percentage of softwoods and hardwoods in an average forest is calculated using data on hardwood and softwood volumes provided by the United States Department of Agriculture (USDA) (2007, 206-208). Data on average productivity classifications for forests

is also taken from the USDA (2007, 160-162). Forests are categorized by the USDA by cubic feet of wood per acre per year (cu. ft.) into one of five different classifications: 120+ cu. ft., 85-119 cu. ft., 50-84 cu. ft., 20-49 cu. ft. and 0-19 cu. ft. A simple average of the extreme values in each productivity class is taken to represent average productivity for forests within that class, while forests with a productivity exceeding 120 cu. ft. are capped at a productivity of 120 cu. ft. The productivity of an average forest in any region of the United States can then be calculated after making these assumptions. Forests on reserved land, or with an average productivity between 0-19 cubic feet per acre per year, are omitted from the stocks of total hardwood and softwood by the USDA, and hence are also omitted from the calculation of average forest productivity.

D.3.3 Table 3: Solar and Wind.

Solar energy calculations based on data from the National Renewable Energy Laboratory. The data can be downloaded from http://www.nrel.gov/gis/data_solar.html. We use the annual average direct normal irradiance for the lower 48 states and Hawaii PV 10km Resolution 1998 to 2009. The data are originally in kWh/m²/day. We transform them to W/m² multiplying by 1000 to get Watts and dividing by 24 to eliminate day from the calculation.

Wind energy calculations are also from NREL and can be downloaded from http://www.nrel.gov/gis/data_wind.html and it is using wind speeds at a height of 50 meters. The exact relation between speed and power is given by the following equation $W = \frac{1}{2}\rho v^3$, where ρ is the density of air and v is the speed. Increasing the height of the tower increases speed. For example increasing the height of the wind tower from 10 to 50 meters increases speed by approximately 25% which in turn would increase power density. Here we also assume wind speeds at 50 meters. To calculate power we assume air density at sea-level and temperature of 15°C which is $\rho = 1.225\text{kg}/\text{m}^3$ (Gipe P., 2004). We assume a cross-sectional area of 1m² sitting on 1m² of land. That is, we assume the radius of the wind turbine is approximately 55% of the area it sit on.

For both solar and wind energy we use GIS to aggregate at the state level calculating the area-weighted average of the cells contained in each state. We then obtain the regional area weighted average using the same regions described above.

The six regions of the United States listed in Table 2 and Table 3 are defined by the USDA (2007, 1) as follows:

- Northeast: Connecticut, Delaware, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont, West Virginia
- North Central: Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, Wisconsin
- Southeast: Florida, Georgia, North Carolina, South Carolina, Virginia
- South Central: Alabama, Arkansas, Kentucky, Louisiana, Mississippi, Oklahoma, Tennessee, Texas
- Rocky Mountain: Kansas, Nebraska, North Dakota, South Dakota, Arizona, Colorado, Idaho, Montana, Nevada, New Mexico, Utah, Wyoming
- Pacific Coast: Alaska, Oregon, Washington, California, Hawaii

E Endogenous Transport Costs

We solve the energy producer's problem in two stages. In the first stage transportation costs are minimized by choosing how much distance to cover by land and how much distance to cover by river. In the second stage energy rents are maximized. The cost minimization problem is given by:

$$\min_{r_1, r_2} cr_1 + \rho cr_2, \text{ subject to } r^2 = r_1^2 - r_2^2 + 2rr_2 \cos \theta \quad (\text{E.1})$$

where r_1 is the distance travelled by land and r_2 is the distance travelled by road. The constraint follows directly from the law of triangles with r_1 being opposite to the angle θ . We

can replace the constraint in the objective function to find the optimal distances travelled by land and by road:

$$r_1^* = \frac{\sin \theta}{(1 - \rho^2)^{1/2}} * r \text{ and } r_2^* = \cos \theta - \frac{\rho \sin \theta}{(1 - \rho^2)^{1/2}} * r. \quad (\text{E.2})$$

If the distance r_2^* is strictly positive, the supplier deviates to the road, otherwise the supplier goes straight to the core. We can solve for the critical value of θ that separates the suppliers that go straight to the core from those who deviate to the road:

$$r_2^* > 0 \text{ if and only if } \theta \leq \cos^{-1} \rho \equiv \bar{\theta} \quad (\text{E.3})$$

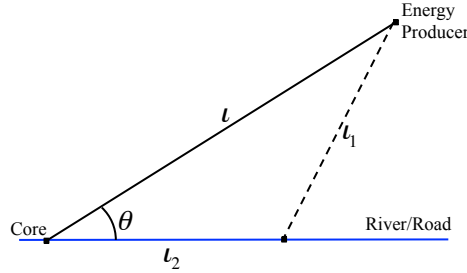


Figure 4: Transport via Road or River

Energy suppliers located at any angle $\theta < \bar{\theta}$ are “close” to the low cost alternative and choose to use it. Since $\rho = \cos(\bar{\theta})$, we know that as $\rho \rightarrow 0$ everyone deviates, since it is so cost effective. Alternatively, as $\rho \rightarrow 1$, the road offers no advantage and no one uses it.

The second part of the energy producer’s problem is to decide whether or not to take its energy to the core. An energy producer situated a distance r from the core and forming an angle θ with the road will go to the core if the net energy supplied to the core is positive; i.e., if there are positive energy rents at this location. Energy supplied by this producer is given by $W^S = \Delta - c(r_1^* + \rho r_2^*)$ Replacing equations (E.2) in the previous equation makes energy rents a function we find $W^S = \Delta - c(\theta)r$ where $c(\theta)$ given by (??).

We depict the exploitation zone in the river and road case in the two panels of Figure 5 assuming $\rho < 1$.

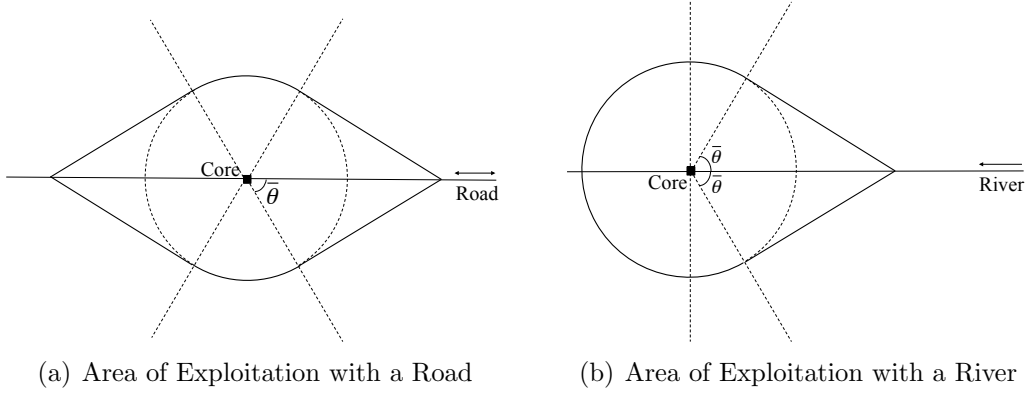


Figure 5: Rivers and Roads

Just as before, the total energy supplied to the core is found by “adding up” all energy rents.

$$W^S = 4 \times \left[\int_0^{\bar{\theta}} \int_0^{r^*} v(\Delta - c(\theta)v) dv d\theta + \int_{\bar{\theta}}^{\pi/2} \int_0^{r^*} v(\Delta - cv) dv d\theta \right]$$

The first integral represents energy coming from suppliers who are close enough to the road to use it in transport. The second integral represents the energy coming from those who travel directly to the core. We have multiplied the integrals by 4 since we are adding up over the quarter circles of $\pi/2$ radians.

Integrating and simplifying gives us a net energy supply much like that we had before:

$$W^S = \frac{1}{3} \frac{\Delta^3}{c^2} g(\rho) \tag{E.4}$$

$$g(\rho) = \pi + 2(\tan(\bar{\theta}) - \bar{\theta}) \geq 0 \tag{E.5}$$

where the function $g(\rho)$ is positive and monotonic $g'(\rho) < 0$, approaches infinity as ρ goes to zero and approaches π as ρ goes to 1.

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